Non-ideal effects in magnetised BNS merger simulations

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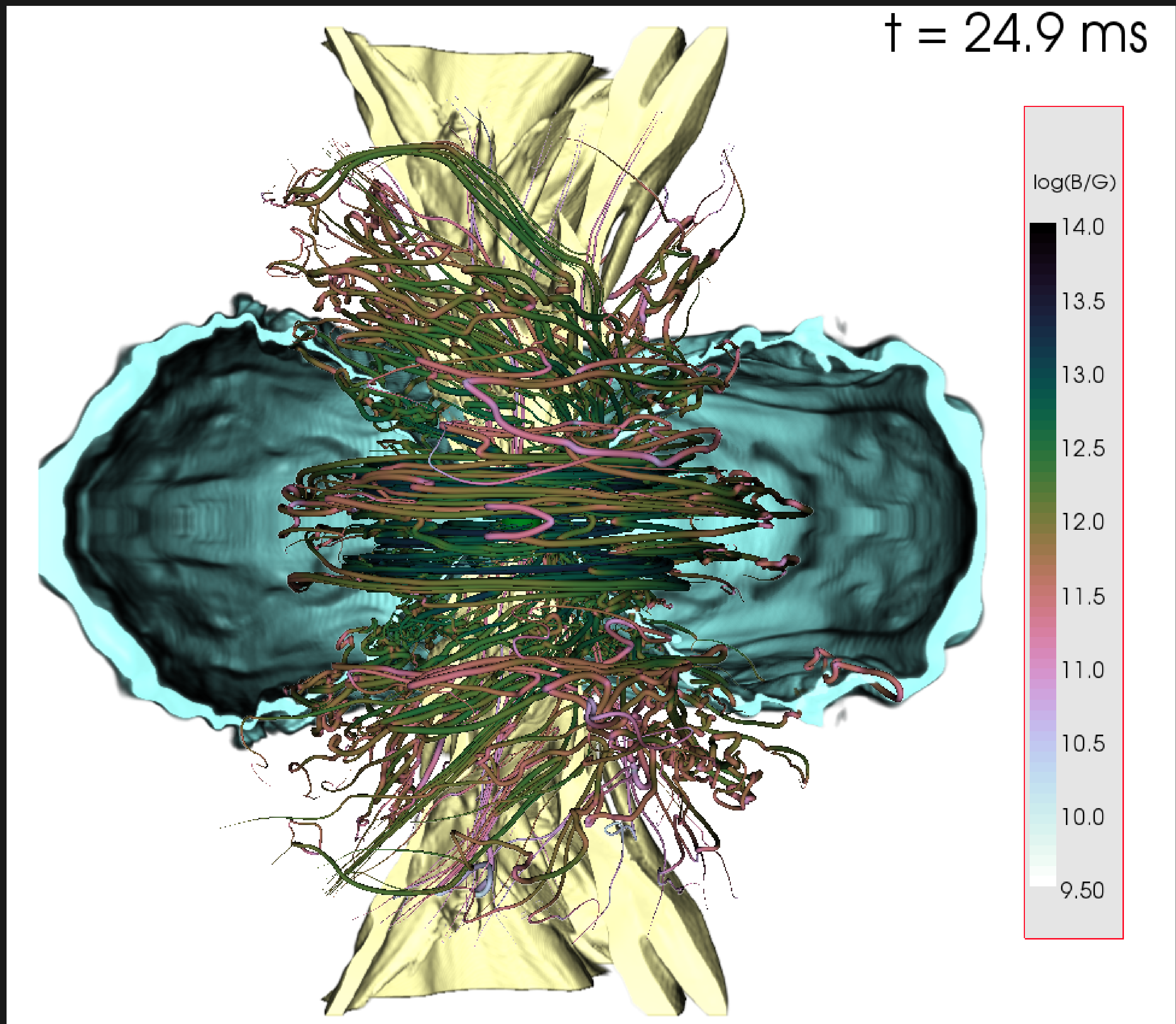
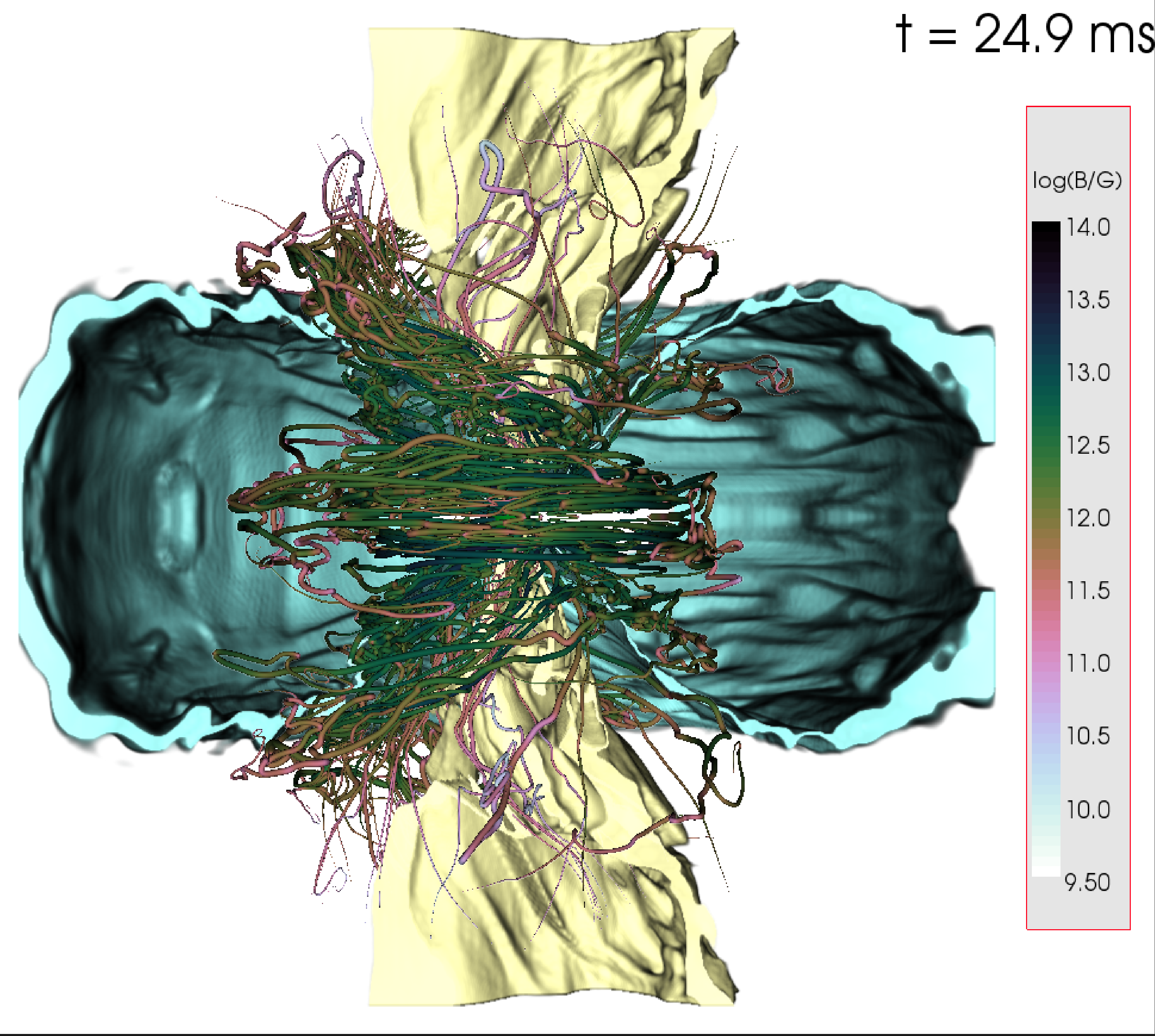
ianhawke.github.io/slides/compose21





Non ideal MHD

Ideal MHD assumes $\sigma \to \infty$ to enforce

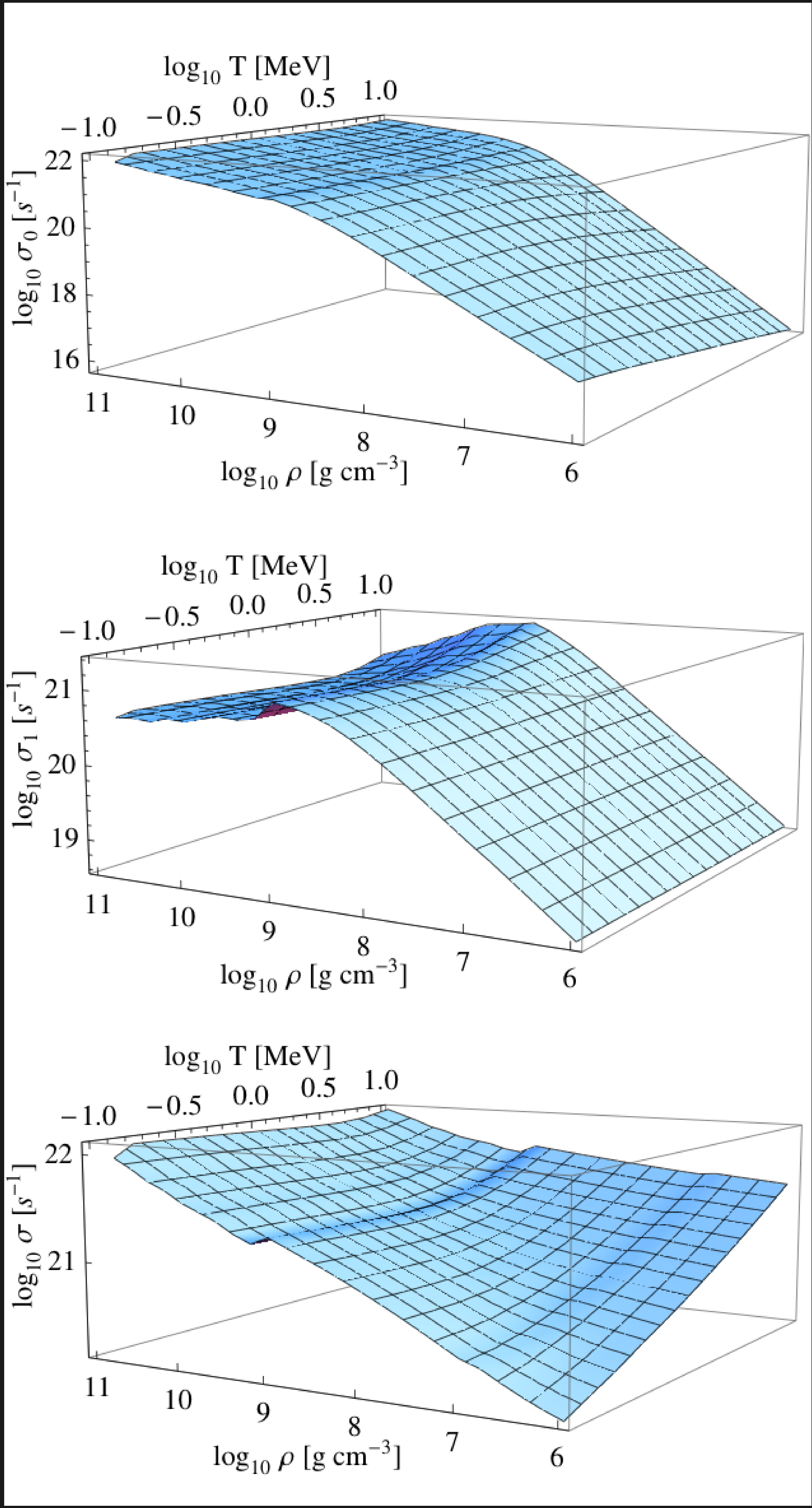
$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}.$$

Detailed calculations show σ drops when

- density drops;
- temperature increases;
- magnetic field increases.

All possible in (post) merger.

Harutyunyan & Sedrakian, 1605.07612



Resistive MHD

Instead impose $e^a \sim \eta j^a$,

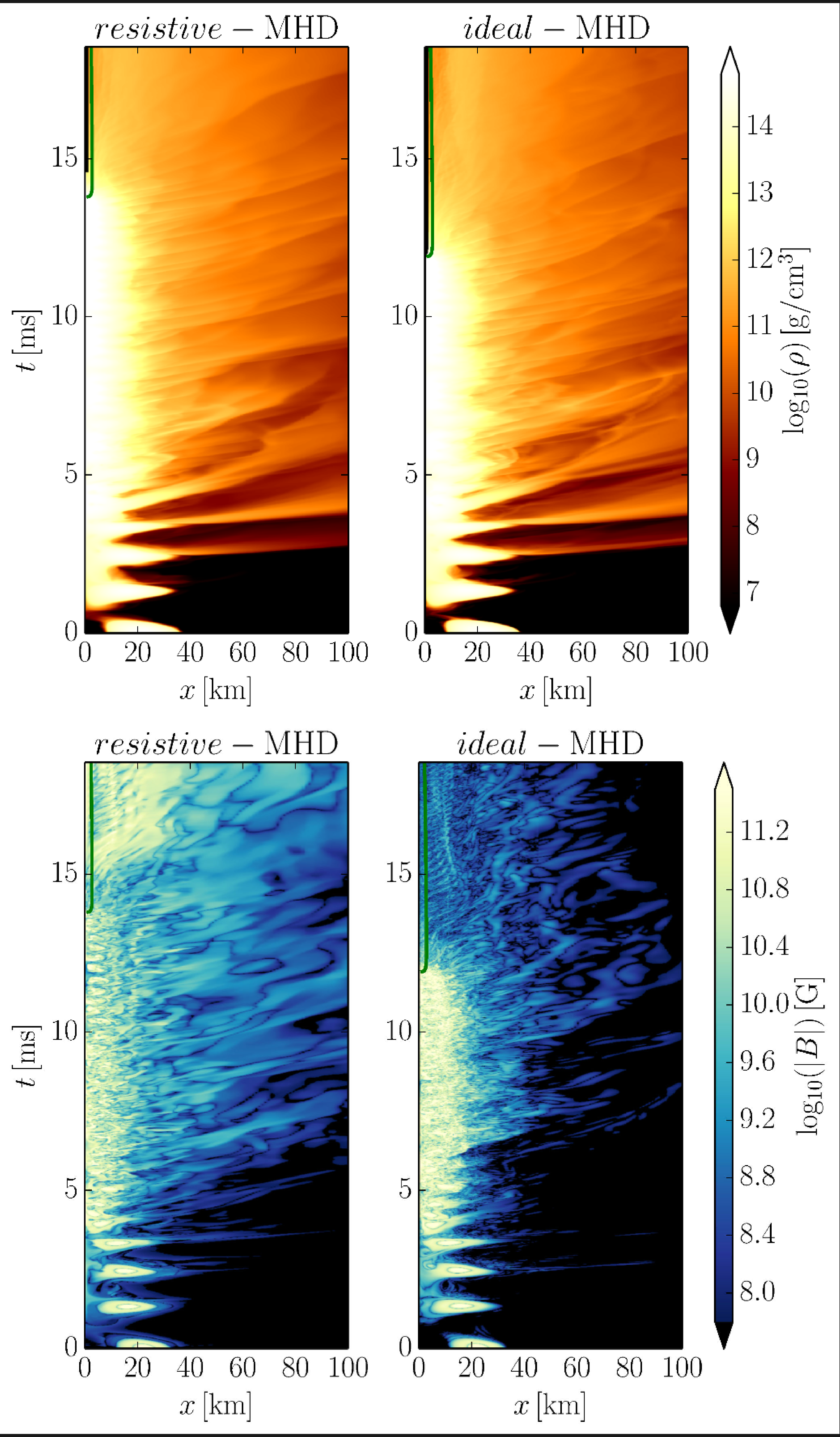
$$\partial_t E_i + \partial_k(\epsilon^{ijk} B_j) \sim \frac{W}{\eta}\left[E_i + \epsilon_{ijk} v^j B^k - (v_k E^k) v_i\right].$$

Simulations done at "large" resistivity, eg

- Ponce et al,
- Dionysopoulou et al,
- Shibata et al.

Numerical *stiffness* as $\eta \to 0$.

Dionysopoulou et al, 1502.02021



LES effects

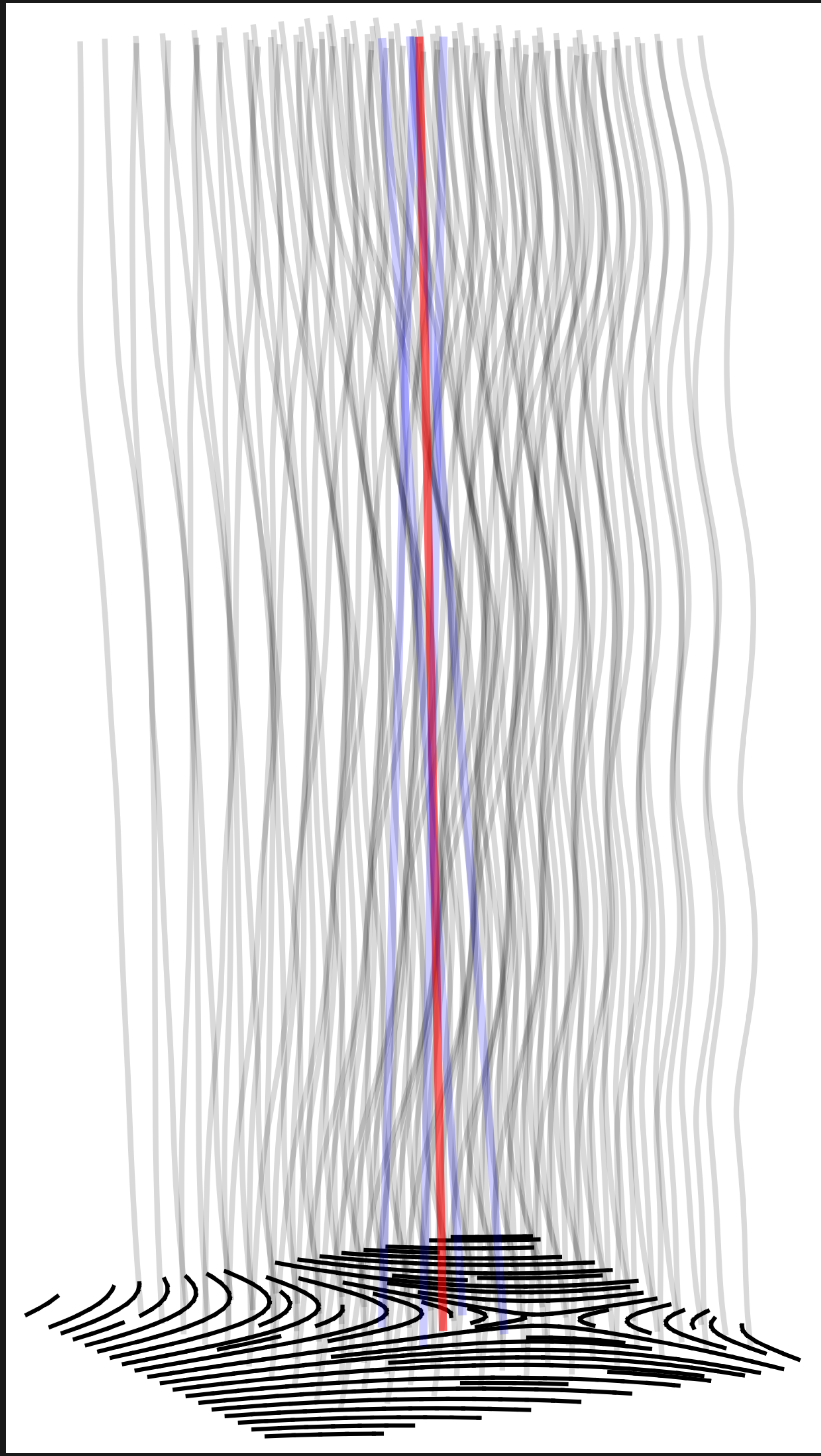
- Even if physics ideal on *micro*-scale...
- Numerics $\implies$ average over $\sim 10 - 100$m scale.

Averaging: $u^a = \langle u^a \rangle + \delta u^a \to$ "non-ideal" subgrid closures.

Averaged currents won't be ideal! Eg

$$\eta \sim \langle -\delta u^b j^a \langle F_{ba} \rangle + u^b j^a \delta F_{ba} \rangle.$$

See eg Viganò et al, or Bucciantini and del Zanna.



Relaxation systems

Toy problem of Liu (see LeVeque):

$$\begin{aligned} \partial_t B + \partial_x E &= 0, \\ \partial_t E + a \partial_x B &= \eta^{-1} \left(f(B) - E \right). \end{aligned}$$

Use *Chapman-Enskog* expansion $E = f(B) + \eta E_1$ to get

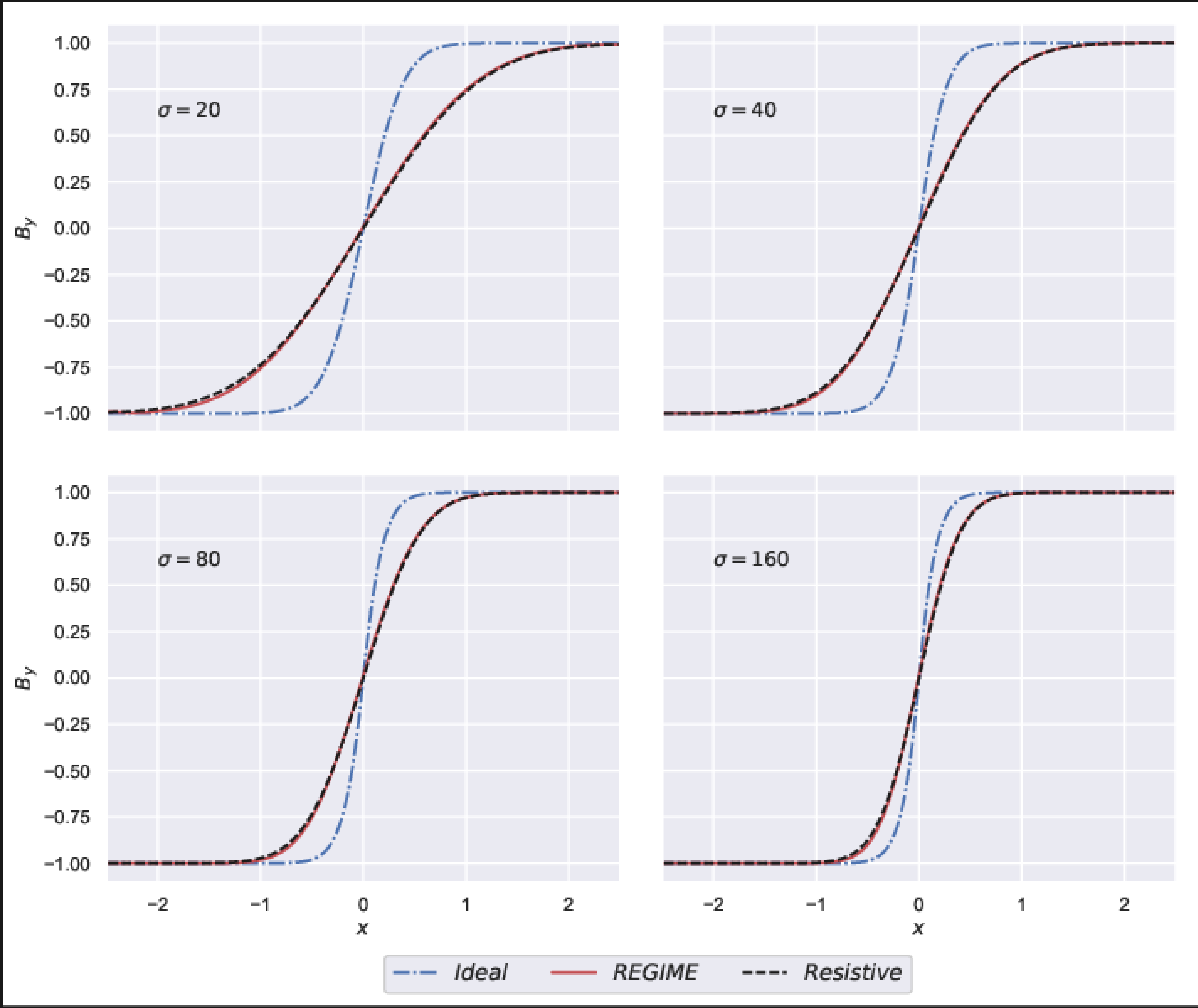
$$\partial_t B + \partial_x f(B) = \eta \partial_x \left(\left[a - \left(f' \right)^2 \right] \partial_x B \right).$$

- Smaller system, so cheaper.
- Source now $\propto \eta$, not η^{-1}: stiffness gone.
- Diffusive correction: shocks, timestep, stability issues.

Current sheet

- Standard error-function diffusive solution;
- Use "small" conductivity $\sigma = \eta^{-1}$: hard test for expansion;
- Extremely good agreement with resistive MHD (using IMEX);
- Expected convergence with conductivity.

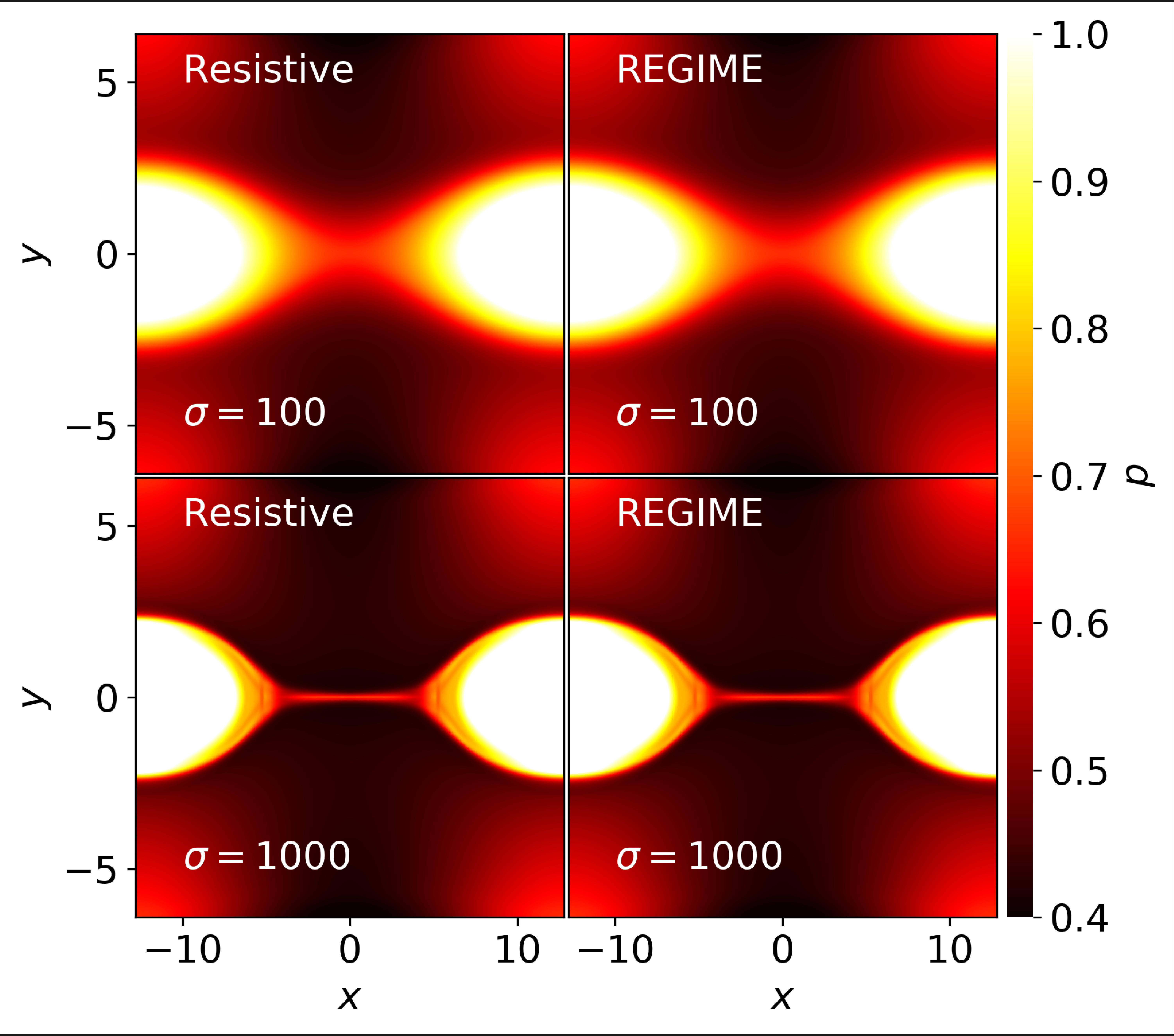
Wright & Hawke, 1906.03150



Reconnection

- Resistivity leads the magnetic "islands" to connect;
- The width of the connection is $\propto \eta = \sigma^{-1}$;
- Extremely good agreement with resistive MHD;
- Expected convergence with conductivity.

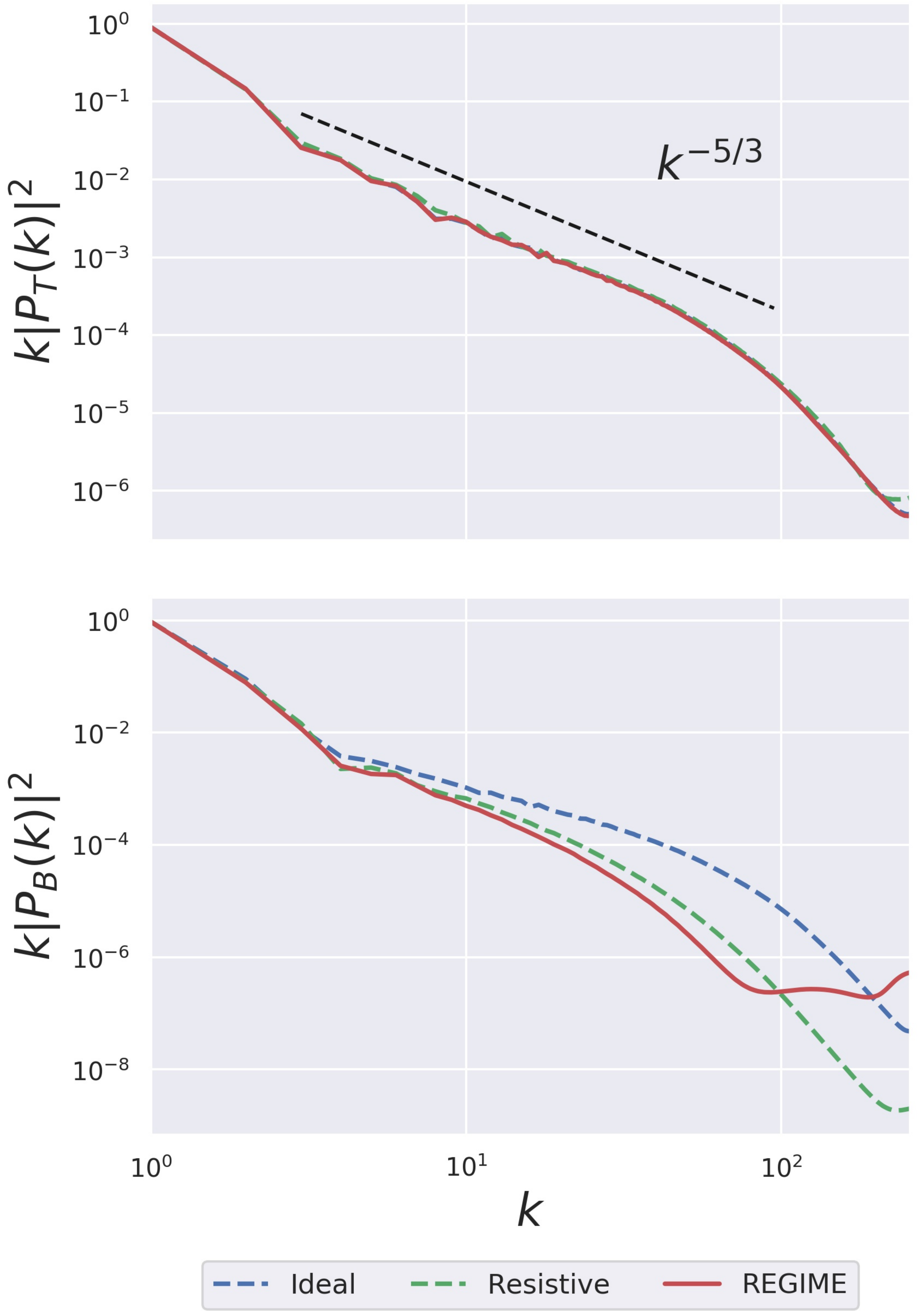
Wright & Hawke, 1906.03150



Kelvin-Helmholtz Instability

- Magnetic energy cascades to all scales;
- Good agreement with resistive MHD at larger scales;
- The source term approach is *not* capturing subgrid behaviour.

Wright & Hawke, 1906.03150

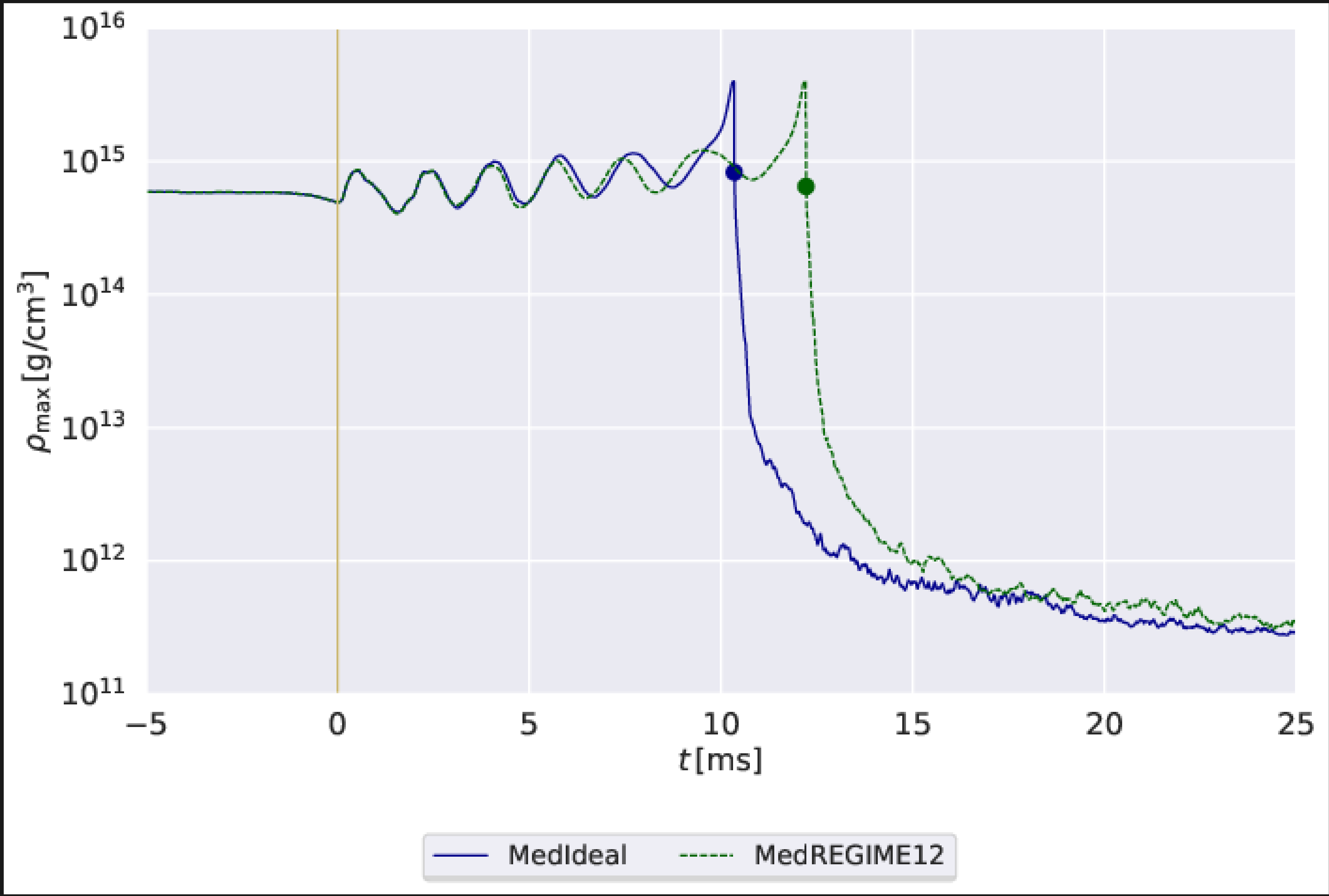


Performance

- Many factors faster than full model.
- For neutron stars want $\sigma \gtrsim 10^{12}$.

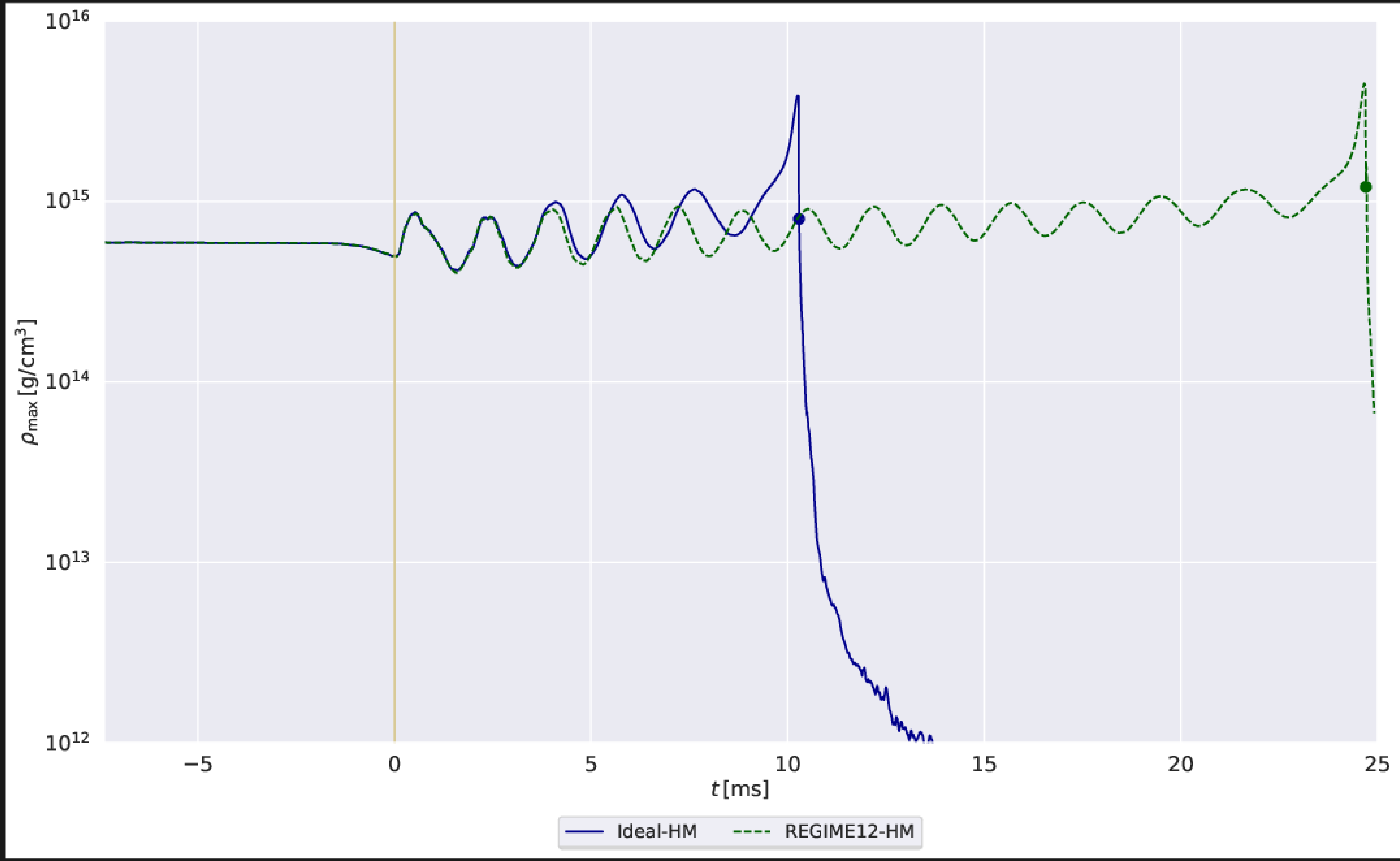
$B \sim 10^{12}\mathrm{G}$

- Kawamura et al, $M \sim 1.6 M_{\odot}$, gamma-law EOS;
- Resistive case $\sigma \simeq 2 \cdot 10^{17}\ \mathrm{s}^{-1}$;
- Qualitative results are similar;
- Post-merger collapse *delayed* by resistivity.



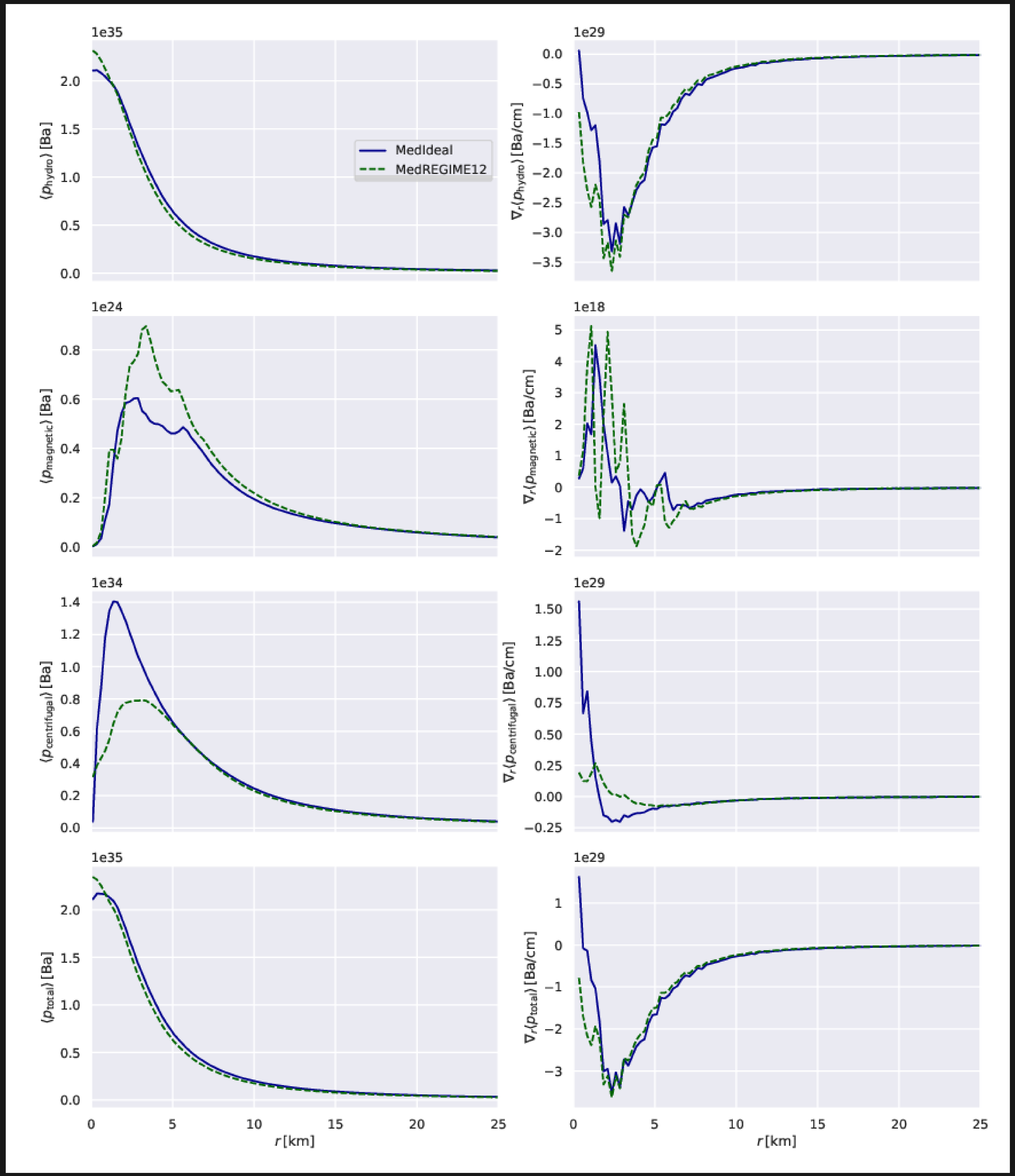
$B \sim 10^{16}\text{G}$

- Helicity timescale $\sim \eta|B|$;
- Resistive case $\sigma \simeq 2 \cdot 10^{17}\ \text{s}^{-1}$;
- Collapse *massively delayed* by resistivity;
- Suggests helicity evolution key?



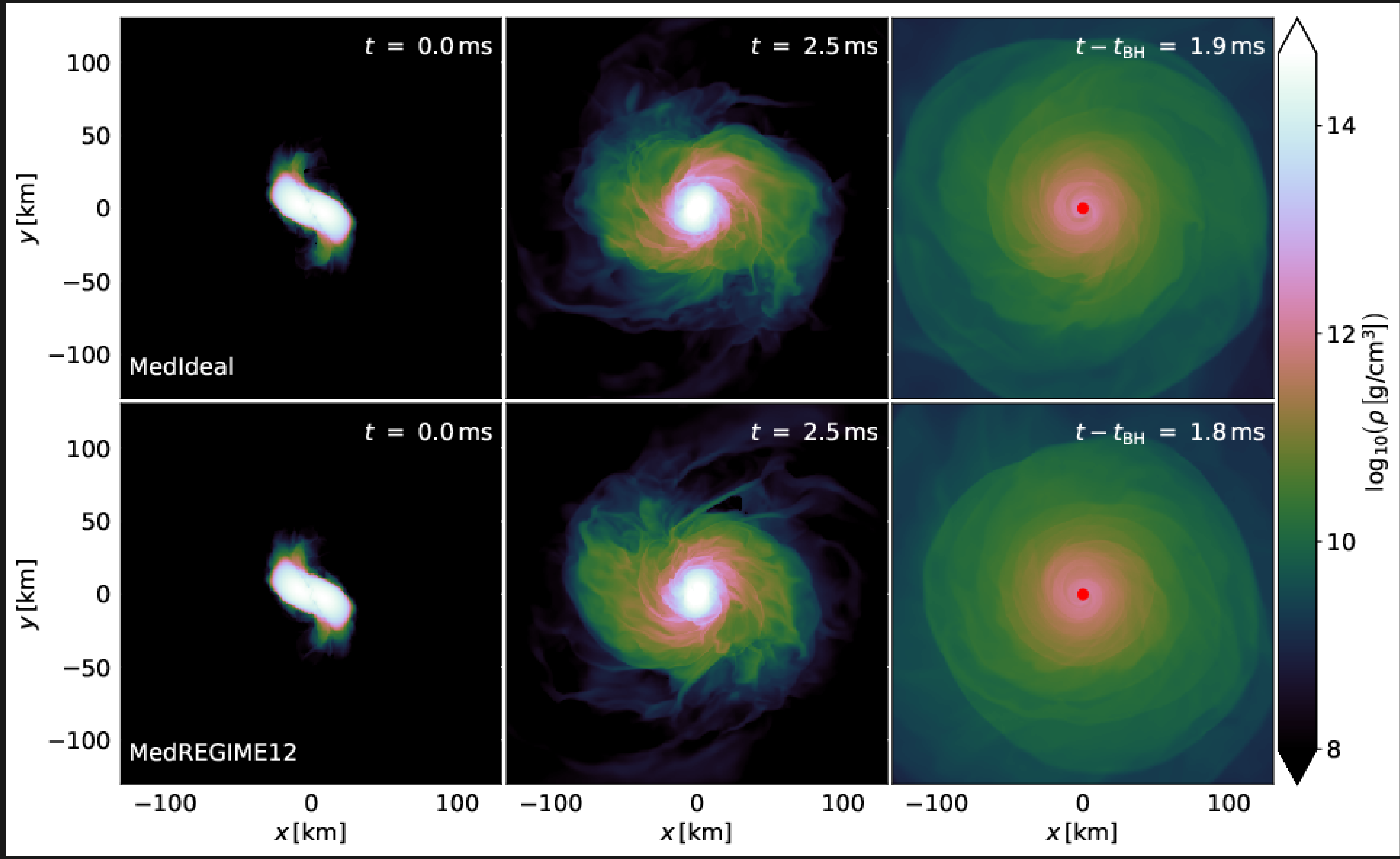
Pressure support

- Angle-averaged slices;
- ρ, e unchanged: energy not put into heat;
- Magnetic field differences larger;
- Effective "centrifugal pressure" main difference;
- Suggests: resistivity re-orders field (helicity), reduces turbulent magnetic drag, increases rotation, delays collapse.



Summary

- BNS mergers often *nearly* ideal.
- Including only leading order non-ideal effects
  - can avoid stiffness problems;
  - can show qualitative physical changes.
- How to capture physical effects on average at this order?



The general case

GR has source terms. Schematically

$$\partial_t \mathbf{q} + \partial_i \mathbf{f}^{(i)}(\mathbf{q}, \overline{\mathbf{q}}) = \mathbf{s}(\mathbf{q}, \overline{\mathbf{q}}),$$
$$\partial_t \overline{\mathbf{q}} + \partial_i \overline{\mathbf{f}}^{(i)}(\mathbf{q}, \overline{\mathbf{q}}) = \eta^{-1} \overline{\mathbf{s}}(\mathbf{q}, \overline{\mathbf{q}}).$$

Chapman-Enskog expansion still works:

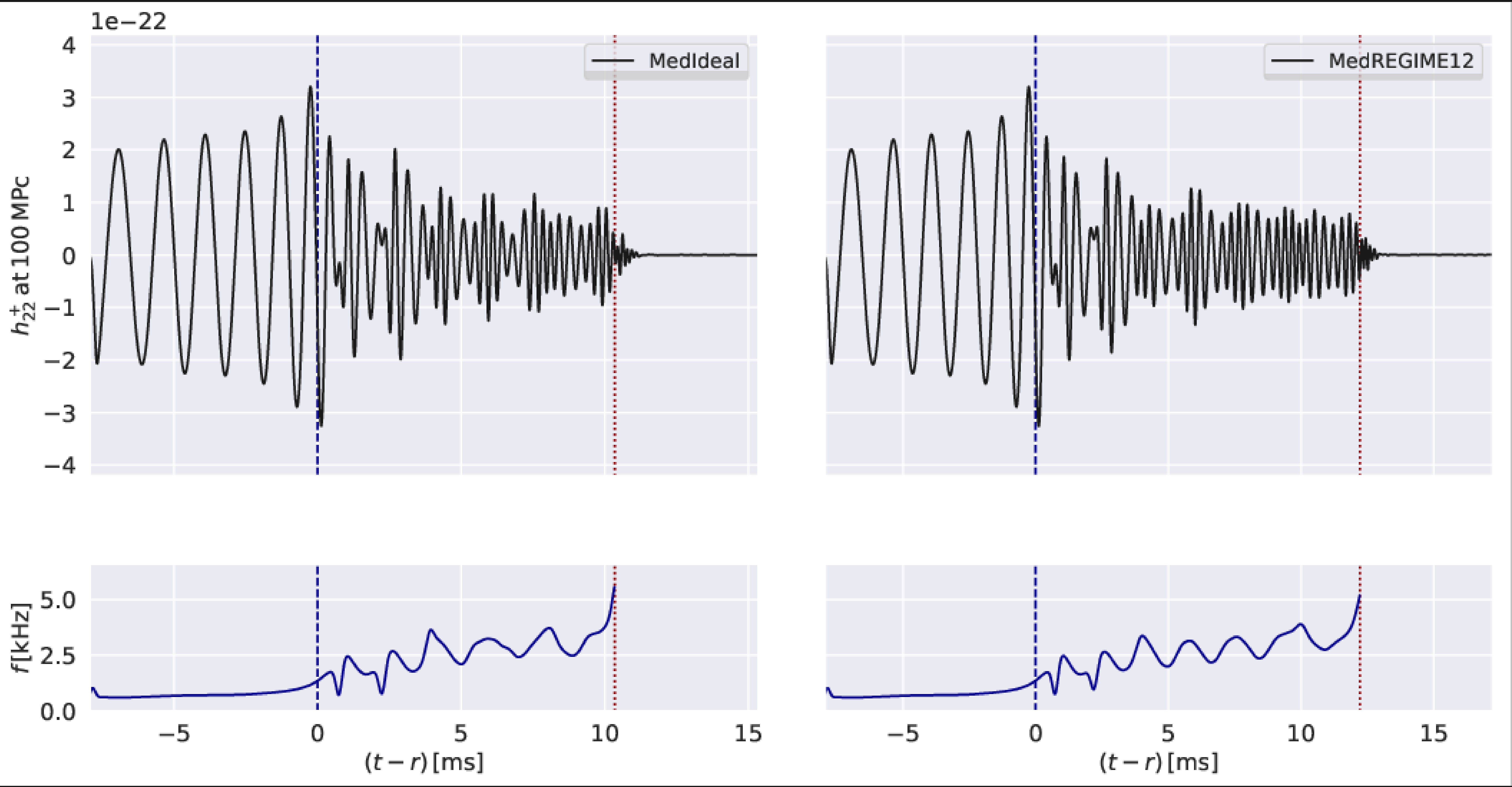
$$\partial_t \mathbf{q} + \partial_i \left(\mathbf{f}_0^{(i)} + \eta \mathbf{F}^{(i)} \right) = \mathbf{s}_0 + \eta \left(\mathbf{S} + \partial_i \mathbf{D}^{(i)} \right).$$

With $\mathbf{D}^{(i)} \sim A^{ij} \partial_j \mathbf{q}$ we still have diffusive correction.

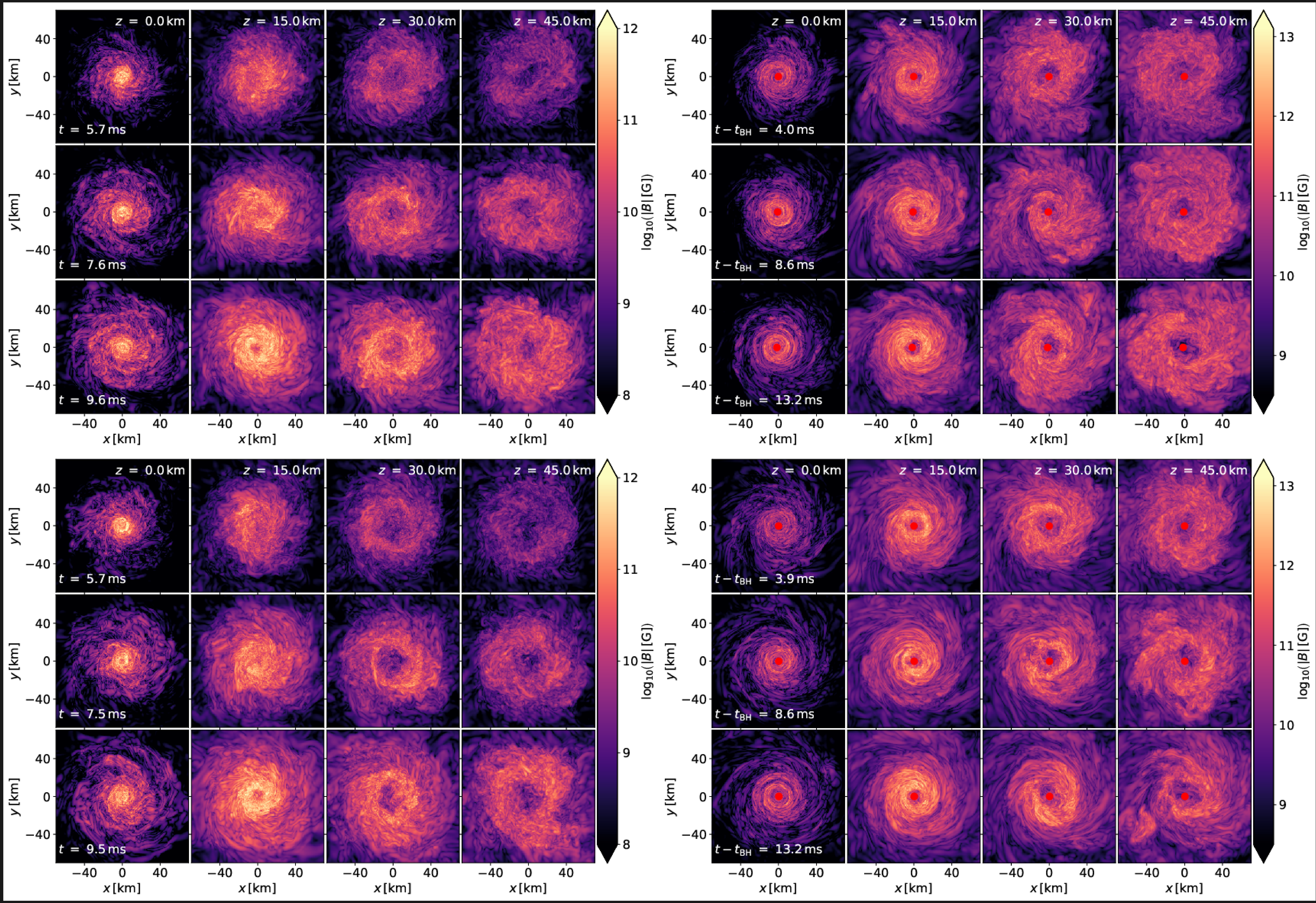
GWs

Resistivity damps modulation faster.

Effect of delayed collapse, not cause.



$|B|$

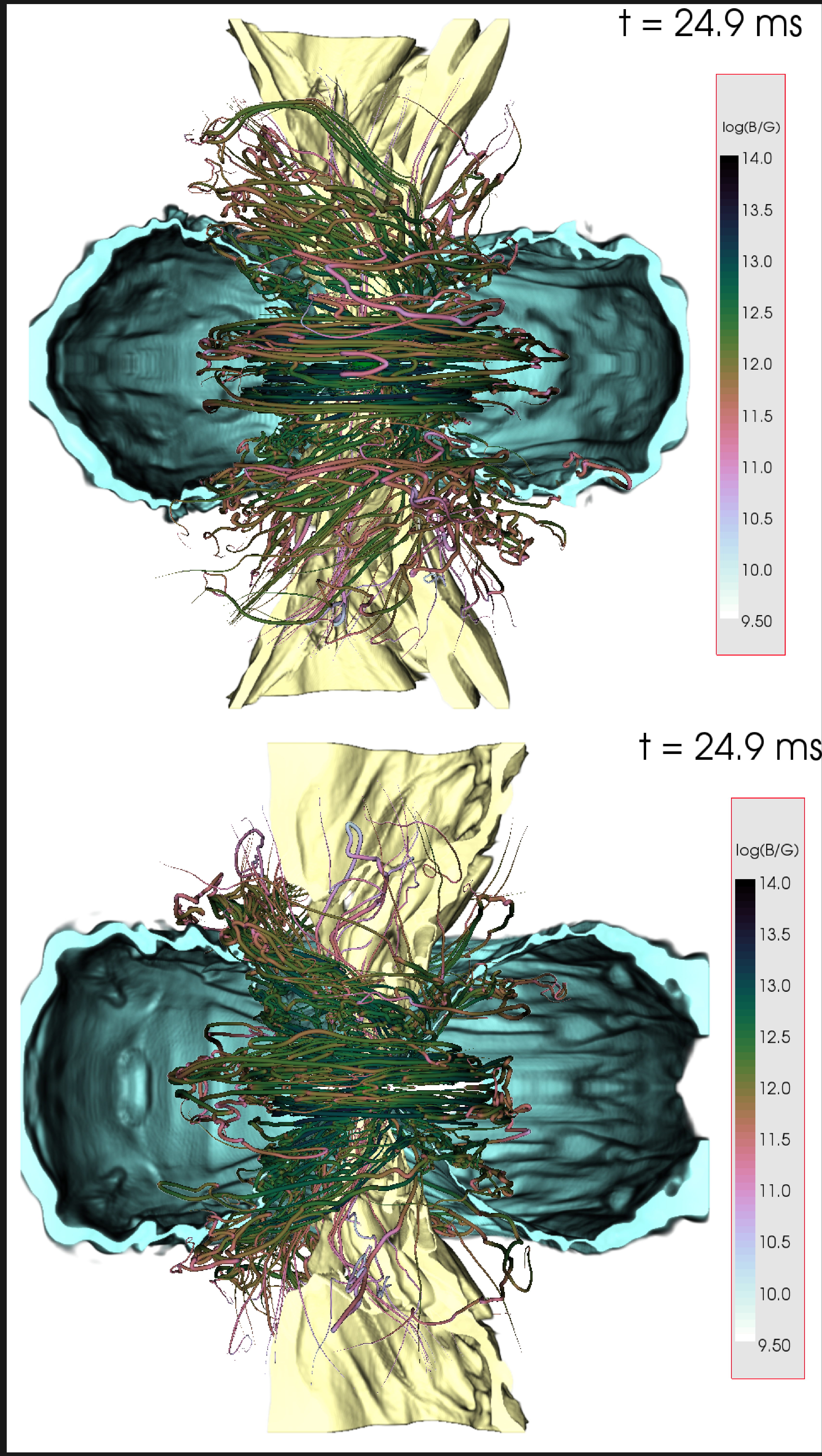


z = 0.0 km
z = 15.0 km
z = 30.0 km
z = 45.0 km
t = 5.7 ms
t = 7.6 ms
t = 9.6 ms
t = 5.7 ms
t = 7.5 ms
t = 9.5 ms
t − t_BH = 4.0 ms
t − t_BH = 8.6 ms
t − t_BH = 13.2 ms
t − t_BH = 3.9 ms
t − t_BH = 8.6 ms
t − t_BH = 13.2 ms
y [km]
x [km]
−40
0
40
log10(|B| [G])
8
9
10
11
12
13

Centrifugal pressure

Field Structure

- Resistivity changes the magnetic field structure;
- Post-merger the field is "turbulent", messy;
- Resistivity re-orders the field, reducing "drag";
- Increase in rotational effective pressure seen, delaying collapse.



Israel-Stewart toy model

Extreme simplification of Israel-Stewart:

$$\partial_t T + \partial_x q = 0,$$
$$\tau_q \partial_t q = \tilde{q}(T) - q, \qquad \tilde{q} = -\lambda \partial_x T.$$

Chapman-Enskog for $\tau_q \ll 1$:

$$\partial_t T = \lambda \partial_{xx} T - \tau_q \lambda^2 \partial_{xxxx} T.$$

- Retention-diffusion equation.
- Stability limits asymptotically stricter from $\partial_x^{(4)}$ term.