

The nEoS sets from hadron physics alone



Felipe J. Llanes-Estrada
& Eva Lope-Oter
(earlier collaborators,
Mark Alford & Andreas Windisch)

25/Feb/2021

COMPOSE2021 workshop
at Barcelona's avatar

Prerecorded talk available at
<https://youtu.be/5FGpVDpD8kU>

Outline

Motivation: modified gravity

The nEoS sets at Complutense

Upcoming work

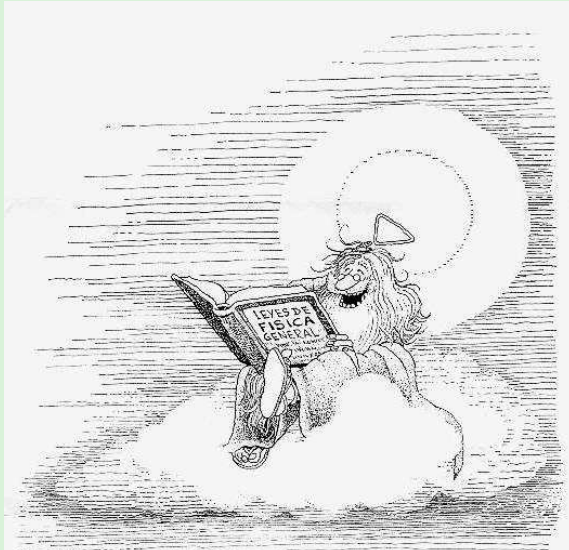
Conclusions

Testing Einstein's GR and constraining modifications

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- ▶ Very well tested at solar-system (weak field)
- ▶ Well tested in binary pulsars (weak field)
- ▶ Tests starting near black holes and N^* mergers (strong field)
- ▶ Gravitational wave propagation

$\frac{c_g - c}{c} < 10^{-15}!!$ but Einstein failed Unified Field Theory...



(As this cartoon
by Quino
illustrates)

But all are tests of the LHS

$$G_{\mu\nu} = \cancel{\frac{8\pi G}{c^4}} \cancel{T}_{\mu\nu}$$

Need to assess the Neutron Star interior!

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$$G_{\mu\nu} = \cancel{\frac{8\pi G}{c^4}} \cancel{T}_{\mu\nu}$$

Need to assess the Neutron Star interior!

Distinguishing Nuclear matter- from Gravitational- effects

The strong equivalence principle difficults assigning corrections to either T or G

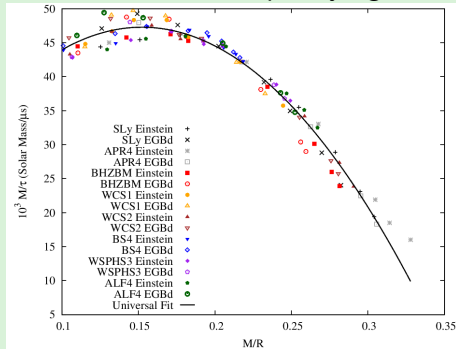
Example: cosmological constant

$$G_{\mu\nu} + \lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Same problem in neutron stars

So here an example from home

N-Star oscillation frequency against mass



Can displace the curve with
both EOS and theory

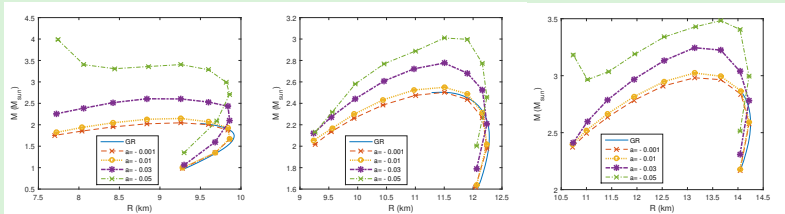
EITHER test the EoS

OR test GR

but not both

(Blázquez-Salcedo, González-Romero
and Navarro-Lérida @ UCM)

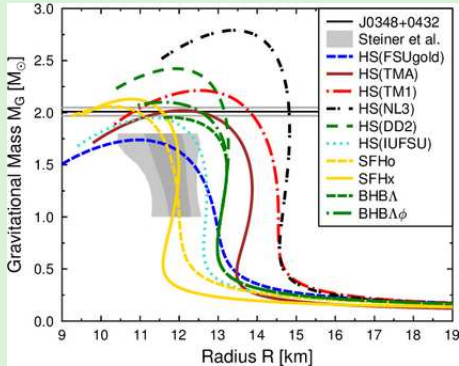
$R + aR^2$ gravity: N-star mass increased for large a



- State equations of Hebeler *et al.* APJ**773**:11 (2013)
- Matching to exterior Schwarzschild (careful: lot of energy there)
- Find heavier stars
- Can also find twin branches from modified gravity

(M. Aparicio Resco et al., Phys. Dark Universe 2016; also works by Odintsov and coll. and Yazadjev & Doneva)

But same effect upon changing the EoS



<https://astro.physik.unibas.ch/people/matthias-hempel/equations-of-state.html>

Therefore, need to control Hadron input

What is $T_{\mu\nu}$ in a neutron star?
(purely from hadron theory)

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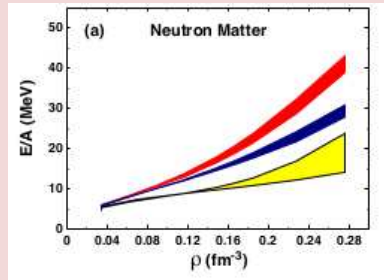
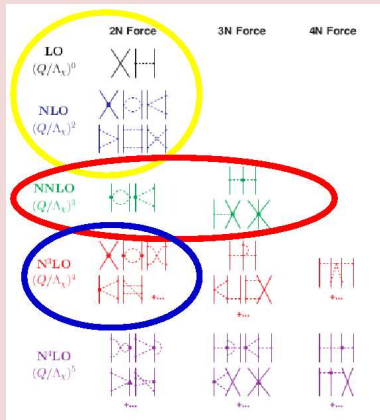
Conclusions

René Descartes taught us



The first was never to accept anything for true which I did not clearly know to be such; that is to say, carefully to avoid precipitancy and prejudice, and to comprise nothing more in my judgment than what was presented to my mind so clearly and distinctly as to exclude all ground of doubt.

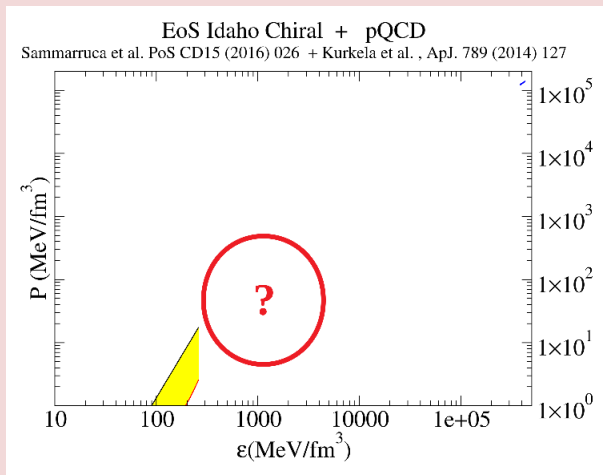
NNLO+ part of N^3LO



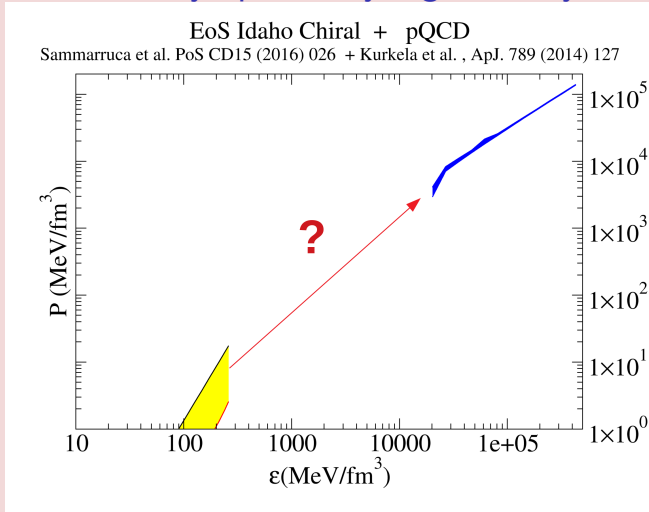
From N^3LO only 2-body part
(to add 3-body, need to refit triton)

Sammarruca *et al.* (INFN-Idaho) Proc. of Science Chiral Dynamics 15 026 (2016)

But the bulk of a neutron star has higher density

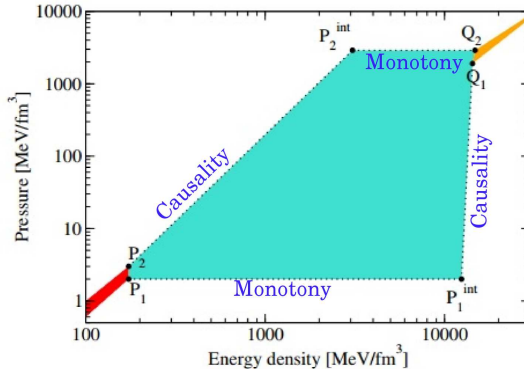


4) EoS from asymptotically high density with pQCD

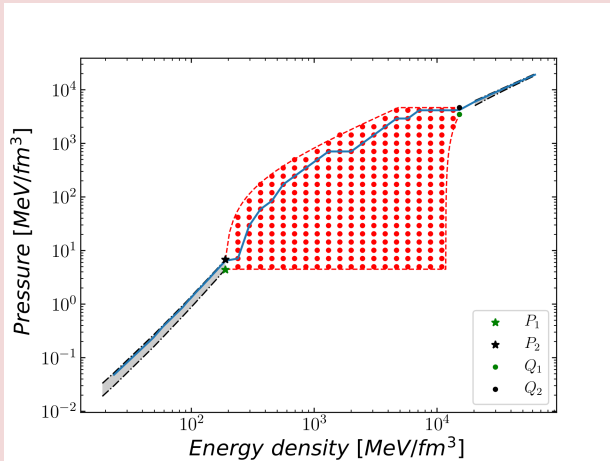


Also A.Kurkela et al., PRD **81** (2010) 105021

Maximum allowed region from first principles

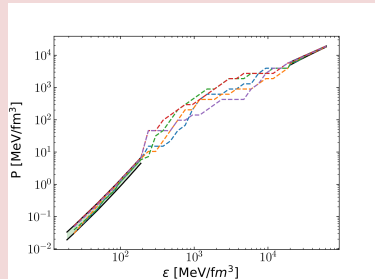
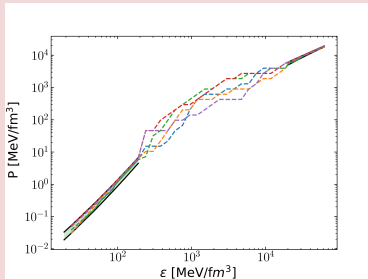


Interpolation with Von Neumann's rejection



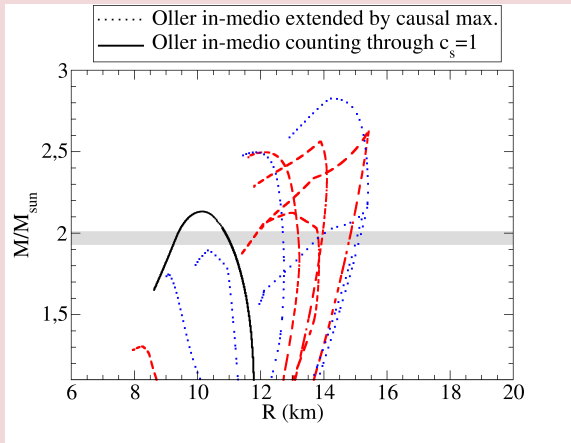
At all points, $c_s \in [0, 1]$

Outcome: bands depending on low-ChPT input

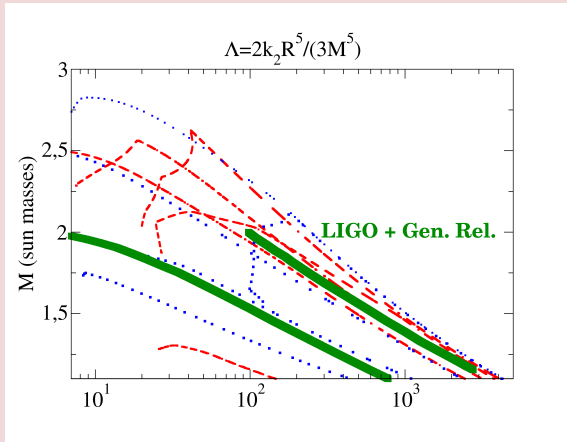


Low P : ChPT High P : pQCD

Example use: mass-radius diagram in GR



Example use: Tidal deformability



(Green band from aLIGO GW170817 + General Relativity)

Website <http://teorica.fis.ucm.es/nEoS>

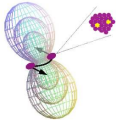
teorica.fis.ucm.es/nEoS/

Not secure teorica.fis.ucm.es/nEoS/

nEoS

neutron matter Equation of State

- You can find **Equations of State for neutron stars** constrained by basic principles and generated with **Hadron Physics input alone** (low-density: Chiral Perturbation Theory for neutron matter; high-density pQCD; intermediate density: monotonicity and causality).
- In particular, no Astrophysics nor General Relativity input has been used. Thus, our sets are **less constrained** than others, but also less biased: you may want to use them, for example, if you are thinking of **testing General Relativity or modified theories of gravity** with neutron stars.
- Random sampling of allowed band in the energy density/pressure diagram. Agnostic about exotic phases in the QCD diagram. Phase transitions are in principle allowed.
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- Given for two baryon chemical potentials at which pQCD is matched to the intermediate density region.



Radiation hole in a merger for various reduced masses
Landau & Lifschitz, 4th french ed. pg. 460
rendering by F.J. Llanes-Estrada
 $dI/d\Omega = 1 + 6\cos(\theta)^2 + \cos(\theta)^4$

Quick start: a few sample typical EoS that satisfy all basic constraints. (Ideal for your class or to toy with a new idea.)	Input constraints from low-density (ChPT).	Full sets (about 0.5 MByte each)
Set 1: low-density constrained by Sammarruca et al. @NLO: Matching baryon chemical potential 2.6 GeV Sample EoS 1: Sample EoS 2: Sample EoS 3: Sample EoS 4: Sample EoS 5	Sammarruca et al., NLO Momentum cutoff $\Lambda=450$ MeV $\Lambda=600$ MeV	Tables provided as a .tar.gz file contain 1000 EoS 1a) Matching baryon chemical potential 2.6 GeV EOS_Sammarruca_NLO 1b) Matching baryon chemical potential 2.8 GeV

You find ten sets with several subsets

teorica.fis.ucm.es/nEoS/		
Sample EoS 3: Sample EoS 5:	Sample EoS 4:	
Set 7: low-density constrained by Drischler et al.; Matching baryon chemical potential 2.6 GeV. Sample EoS 1: Sample EoS 3: Sample EoS 5:	Drischler et al. Lower boundary of allowed band Upper boundary of allowed band	Tables provided as a .tar.gz file contain 1000 EoS 7a) Matching baryon chemical potential 2.6 GeV EOS_Drischler 7b) Matching baryon chemical potential 2.8 GeV EOS_Drischler
Matching baryon chemical potential 2.8 GeV. Sample EoS 1: Sample EoS 3: Sample EoS 5:	 	
Set 8: low-density constrained by Holt and Kaiser @ NLO Matching baryon chemical potential 2.6 GeV. Sample EoS 1: Sample EoS 3: Sample EoS 5:	Holt and Kaiser, NLO $\Delta = 450 \text{ MeV}$ $\Delta = 500 \text{ MeV}$	Tables provided as a .tar.gz file contain 1000 EoS 8a) Matching baryon chemical potential 2.6 GeV EOS_Holt_NLO 8b) Matching baryon chemical potential 2.8 GeV EOS_Holt_NLO
Matching baryon chemical potential 2.8 GeV Sample EoS 1: Sample EoS 3: Sample EoS 5:	 	
Set 9: low-density constrained by Holt and Kaiser @ (partly) N2LO Matching baryon chemical potential 2.6 GeV. Sample EoS 1: Sample EoS 3: Sample EoS 5:	Holt and Kaiser (partly) N²LO $\Delta = 450 \text{ MeV}$ $\Delta = 500 \text{ MeV}$	Tables provided as a .tar.gz file contain 1000 EoS 9a) Matching baryon chemical potential 2.6 GeV EOS_Holt_N2LO 9b) Matching baryon chemical potential 2.8 GeV EOS_Holt_N2LO
Matching baryon chemical potential 2.8 GeV Sample EoS 1: Sample EoS 3: Sample EoS 5:	 	
Set 10: low-density constrained by Holt and Kaiser @ (partly) N3LO Matching baryon chemical potential 2.6 GeV. Sample EoS 1: Sample EoS 3: Sample EoS 5:	Holt and Kaiser (partly) N³LO $\Delta = 450 \text{ MeV}$ $\Delta = 500 \text{ MeV}$	Tables provided as a .tar.gz file contain 1000 EoS 10a) Matching baryon chemical potential 2.6 GeV EOS_Holt_N3LO 10b) Matching baryon chemical potential 2.8 GeV EOS_Holt_N3LO
Matching baryon chemical potential 2.8 GeV Sample EoS 1: Sample EoS 3: Sample EoS 5:	 	

Typical EoS: (ϵ, P) in MeV/fm^3

teorica.fis.ucm.es/nEoS/EOSDATA	
Not secure teorica.fis.ucm.es/nEoS/EOSDATA.DIR/Holt/NLO/EOSHolt_2_6_04.dat	
129.98814999999999	0.88627831040382377
138.77703000000000	1.0067072546076774
147.58109999999999	1.1345698052811621
157.42316000000000	1.2865097931456566
166.32536999999999	1.4311267364072799
175.25260000000000	1.5839618105316162
182.23185000000001	1.7069770472764969
192.24266000000000	1.8937927473735807
200.29506000000001	2.0486767668199537
209.39793000000000	2.2320486152791976
217.52753999999999	2.4010001828050611
225.68101999999999	2.5763892392539982
233.89724000000001	2.7599845266127589
283.89724000000012	5.2290906814847204
347.92600012114980	11.467426043511406
426.39548577612896	16.981882641947934
522.56258579971848	37.241366642514407
640.41873140528287	81.670531980728370
784.85556119003115	179.10394798456952
961.86794939495633	265.23146640256567
1178.8028241405937	392.77599160525551
1444.6641028801523	581.65413656966984
1770.4864014658999	581.65413656966984
2169.7930276846600	581.65413656966984
2659.1572683590816	581.65413656966984
3258.8902663275585	861.36001644577516

What you can download

- ▶ A few loose EoS for a toy idea or a class
- ▶ gZipped packages with 1000 sets each for research runs
- ▶ Or source code to do-it-yourself (at your peril)

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Systematic sensitivity

- ▶ Two values of pQCD starting point:

$$\mu_B \simeq 2.6, 2.8 \text{ GeV}$$

- ▶ Several ChPT calculations (Sammarruca *et al.*, Hu *et al.*, Drischler *et al.*, Holt and Kaiser, at available orders of perturbation theory.)
- ▶ Two values of ChPT momentum cutoff:

$$\Lambda = 0.45, 0.5 \text{ GeV}$$

or similar, when available.

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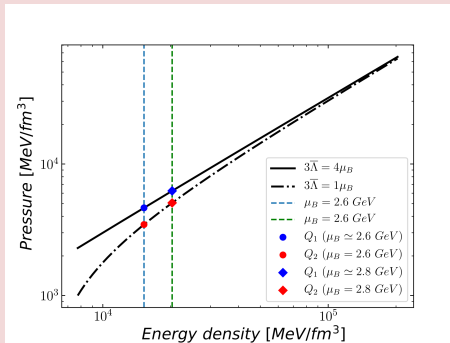
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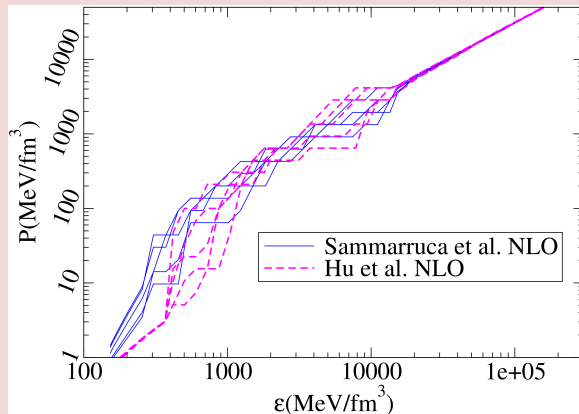
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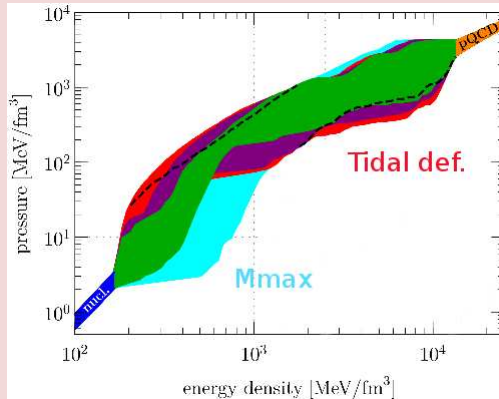
Choice of pQCD starting point



Differences with other sets: nEoS knows no astrophysics



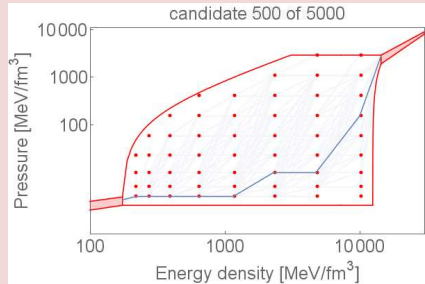
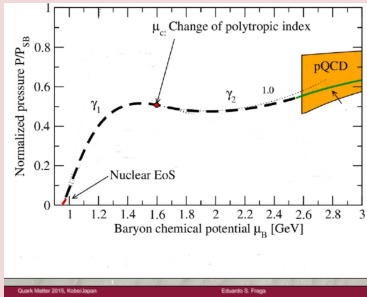
EOS from first principles + mass + tidal constraints



Kurkela *et al.* 1711.02644

If you can assume General Relativity, this work is way tighter
but to constrain beyond GR theories, visit the nEoS webpage!

Interpolation between low and high P



Polytropic $P \propto \rho^\gamma$ vs. nEoS linear interpolation ($\gamma = 0, 1$)
 nEoS more naturally allows for 1st order phase transitions

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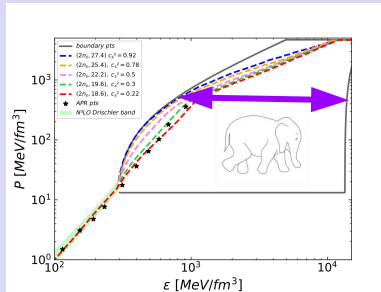
Diagnostic for progress

- Specific latent heat L in a phase transition
 (pure number in natural units)

Substance/transition	L
He-3 superfluid	$1.5 \mu\text{J/mol} = 5.5 \times 10^{-24}$
Ice-water	$79.7 \text{ cal/g} = 3.71 \times 10^{-12}$
Nuclear evaporation	$8 \text{ MeV/A} = 8.5 \times 10^{-3}$
Neutron star matter?	$O(0.1)?$

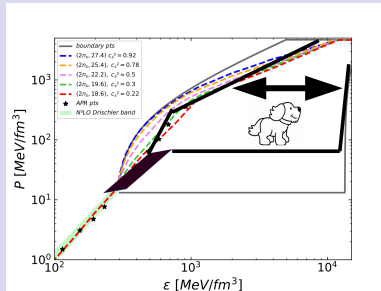
Maximum L in a possible (unknown) phase transition

$$L = \frac{\Delta E}{NM_N} = P_{\text{hadron}} \frac{(\varepsilon_{\text{exotic}} - \varepsilon_{\text{hadron}})}{\varepsilon_{\text{exotic}} \varepsilon_{\text{hadron}}}$$

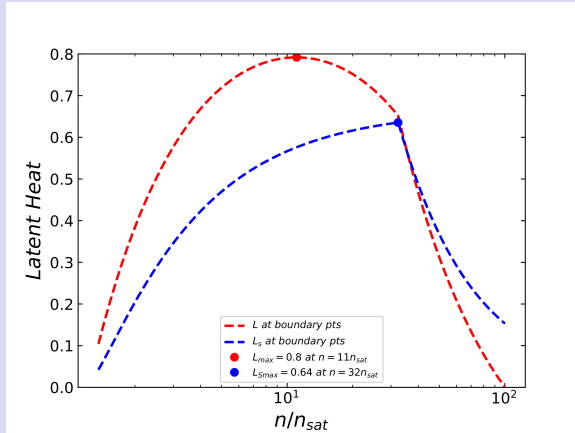


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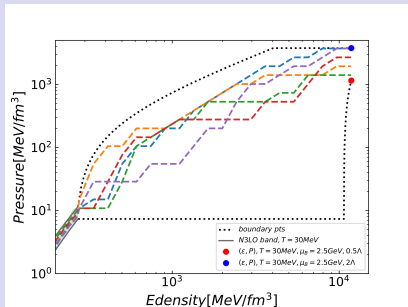
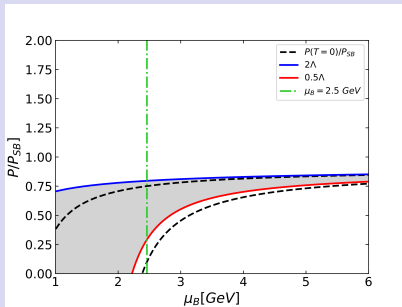
- If the ChPT & pQCD computations improve...



Current status from nEoS: L dependence on transition ε



Finite temperature



Time to reissue a nEoS 2.0?

- ▶ New ChPT computations appearing, finite T , improve sampling...
- ▶ Not well known setup, small return on our effort (Alford et al. JPG 2018)

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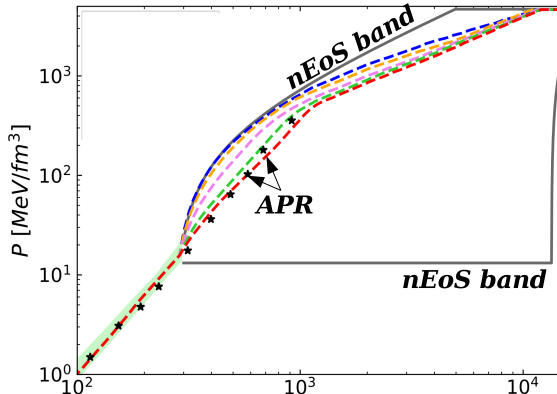
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- If you test modified gravity with a conventional EoS,
you are missing out



Conclusions

- ▶ To fully test the Einstein's equations, we need *the interior* of neutron stars
- ▶ To discriminate between field and matter effects, we need first-principles predictions of $T_{\mu\nu}$, saliently $P(\rho)$
- ▶ We have collected the state of the art information thereof in the nEoS project

Website <http://teorica.fis.ucm.es/nEoS>

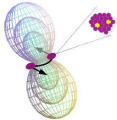
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nEoS

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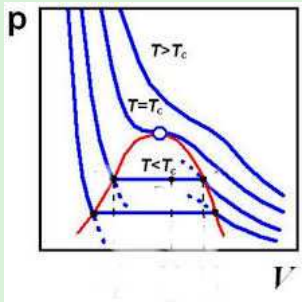


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Landau & Lifschitz, 4th french ed. pg. 460
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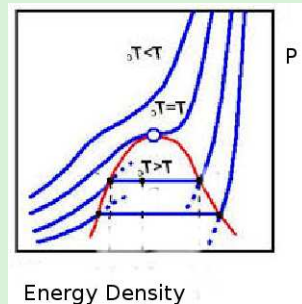
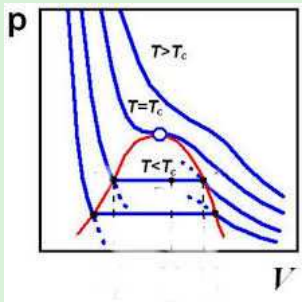
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Backup Slides

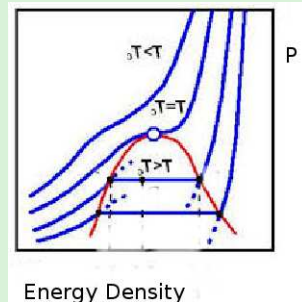
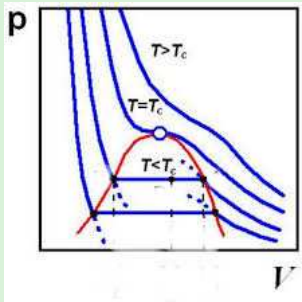
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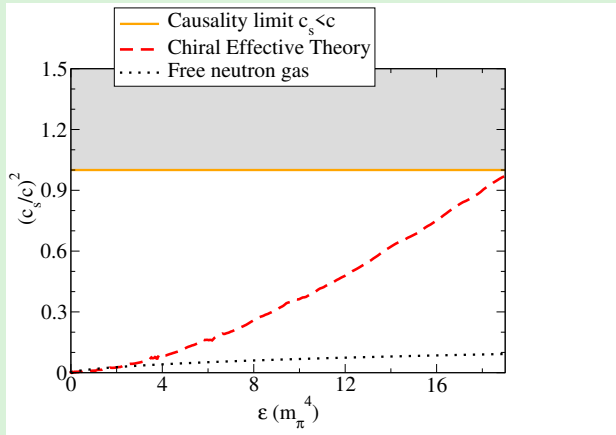


$P(\rho)$ will be **below** whatever Neutron Matter computation yields

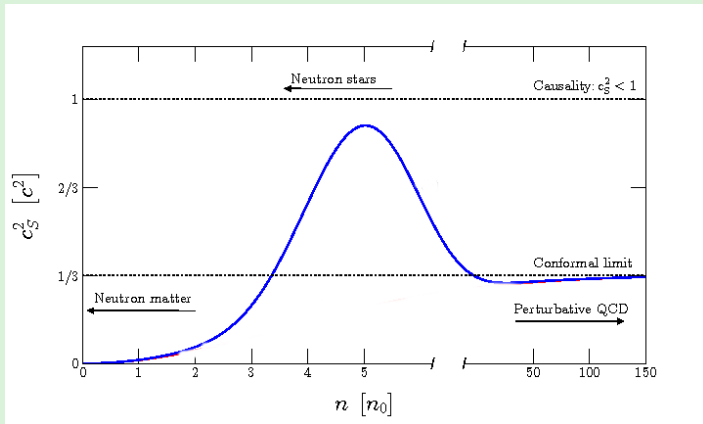


Anything “exotic” beyond neutrons is a softener
 1st order phase transition → latent heat → zero
 derivative

Hardness limited by causality: $c_s = \sqrt{dP/d\rho} \leq c$

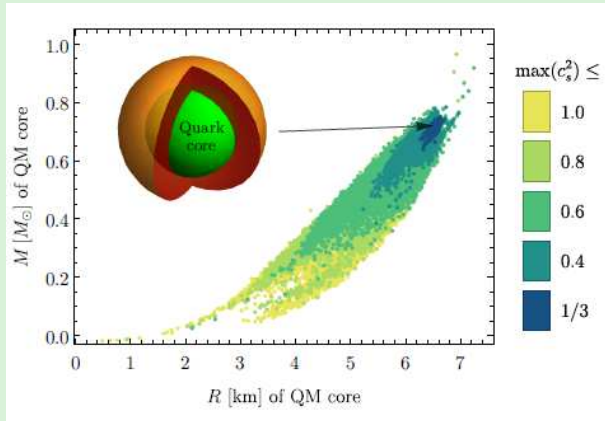


The conformal value is a red herring: $c_s^2 > 1/3$ seems necessary



Graph from McLerran and Reddy, Phys.Rev.Lett. 122 (2019), 122701

Small exception to $c_s^2 > 1/3$ by Helsinki group



Graph from Annala *et al.* arXiv:1903.09121 [astro-ph.HE]