Neutron-star crusts at finite temperature in a multi-component approach

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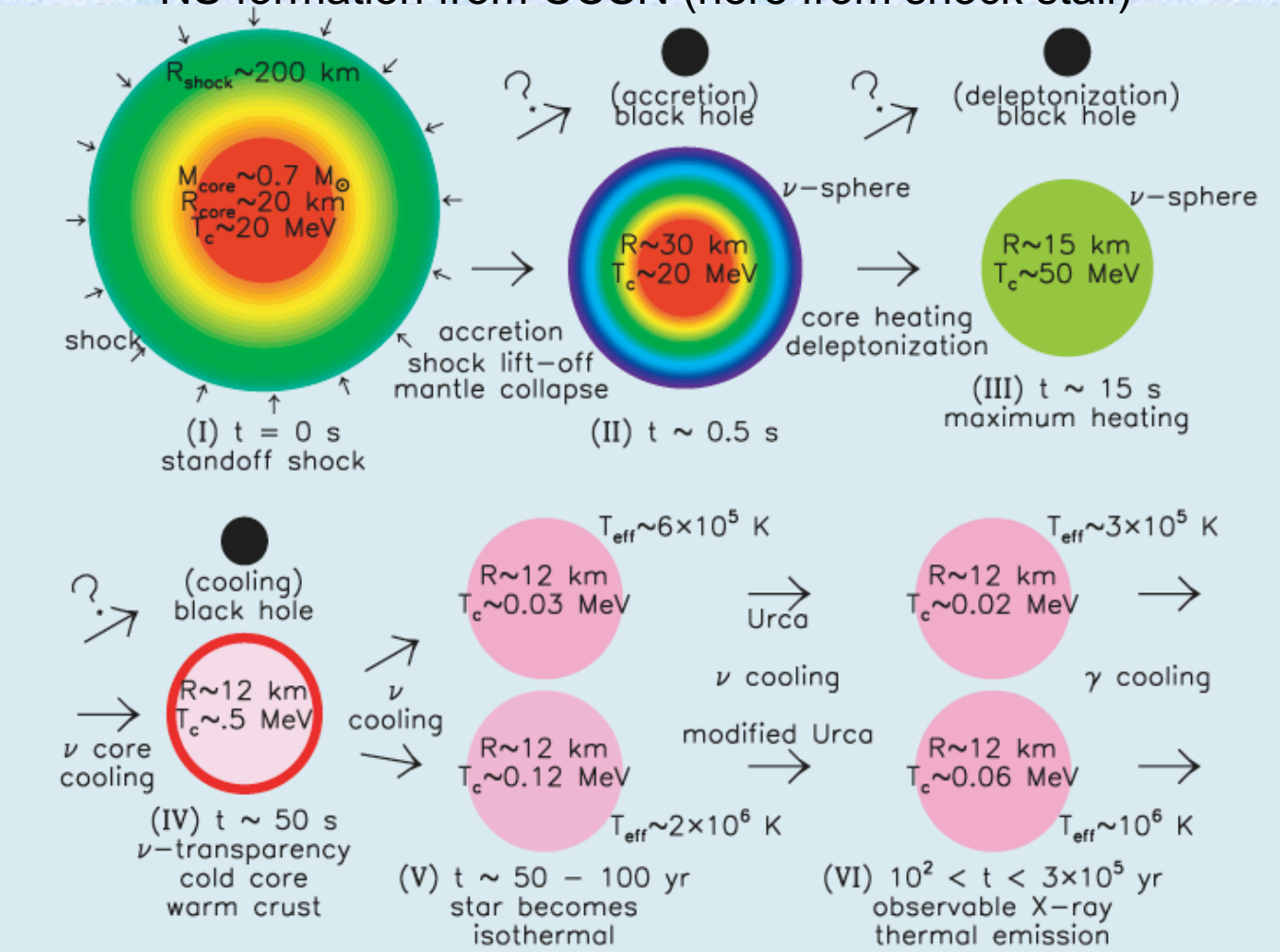
Outline

- Introduction
  - Astrophysical context and motivations
- Model(s) for the crust
  - One-component (OCP) vs Multi-component plasma (MCP)
  - Thermodynamic conditions for crystallization
- Neutron-star (NS) crust at crystallization (recent results)
  - OCP approximation → crystallization temperature T_m
  - MCP approach → composition, impurity parameter Q_{imp}
- Conclusions & outlooks

Astrophysical context

NS formation from CCSN (here from shock stall)

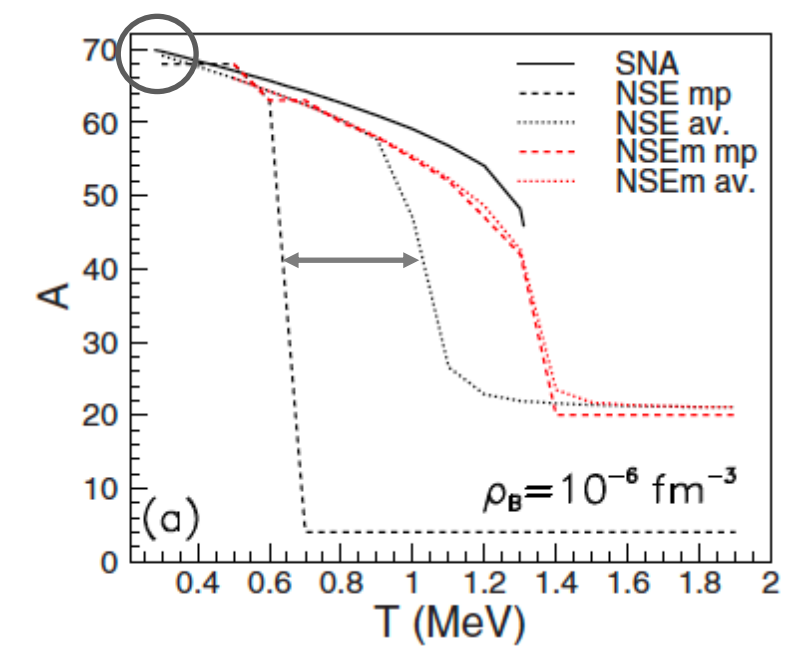


Lattimer & Prakash, Science 304, 536 (2004)

→ NS are born hot from core-collapse SN (T > 1 MeV)

Astrophysical context and motivation

- NS are born hot → "liquid" MCP (ensemble of nuclei)
- As NS cools, MCP distributions become more peaked → OCP
- At T_m, MCP crystallizes → composition frozen



Gulminelli&Raduta, PRC 92, 055803 (2015)

Astrophysical context and motivation

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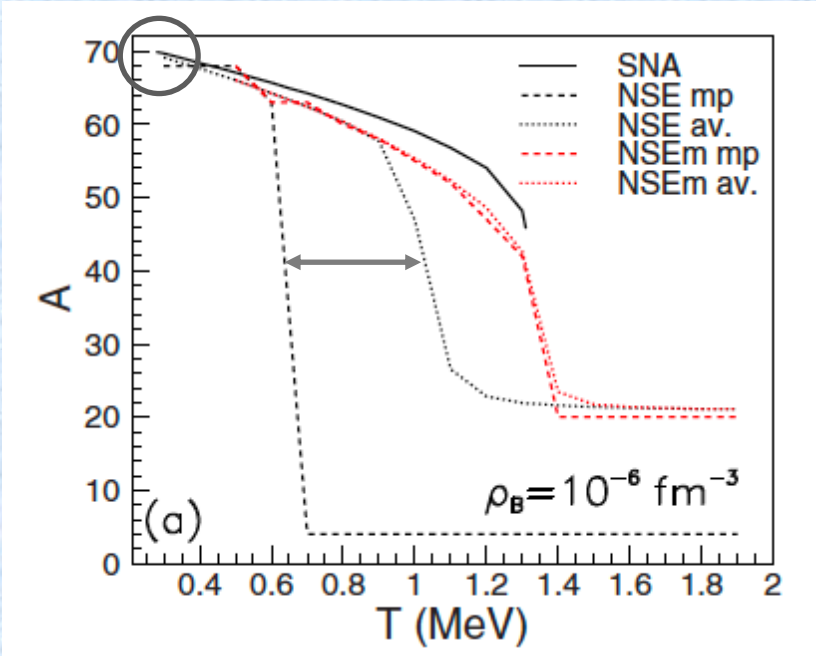
Hp : MCP remains in full equilibrium until the ground state is reached at T=0

but:

- ➢ full equilibrium unlikely to be maintained
- ➢ if cooling rapid enough
  → composition might be frozen at $T > T_m$
  (see e.g. Goriely et al., A&A 531, A78 (2011))
- ➢ other reactions possible below T_m ?
  (see e.g. Potekhin&Chabrier, A&A 645, A102 (2021))



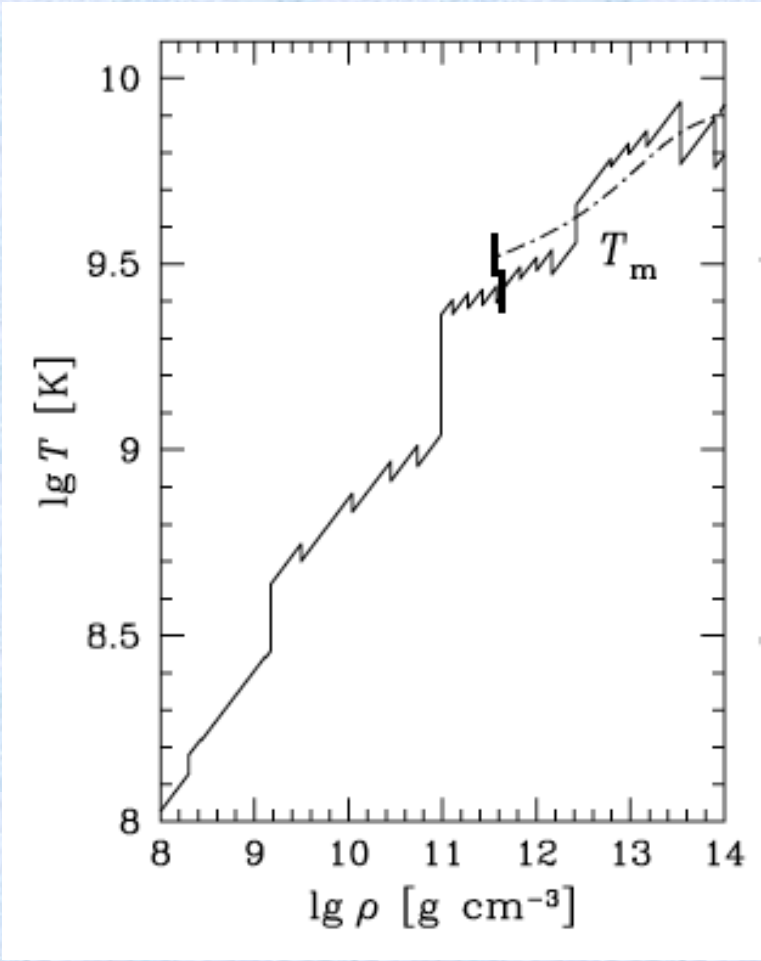
Gulminelli&Raduta, PRC 92, 055803 (2015)

➡ Composition can be different from the T = 0 ground-state one !

Astrophysical context and motivation

- Outer crust : $T_m < 10^9$ K
  → composition close to ground-state OCP
- Inner crust : $10^9 < T_m < 10^{10}$ K
  → situation less clear

Co-existence of nuclear species → "**impurities**"



Haensel, Potekhin, Yakovlev, "*Neutron Star 1*" (Springer 2007)

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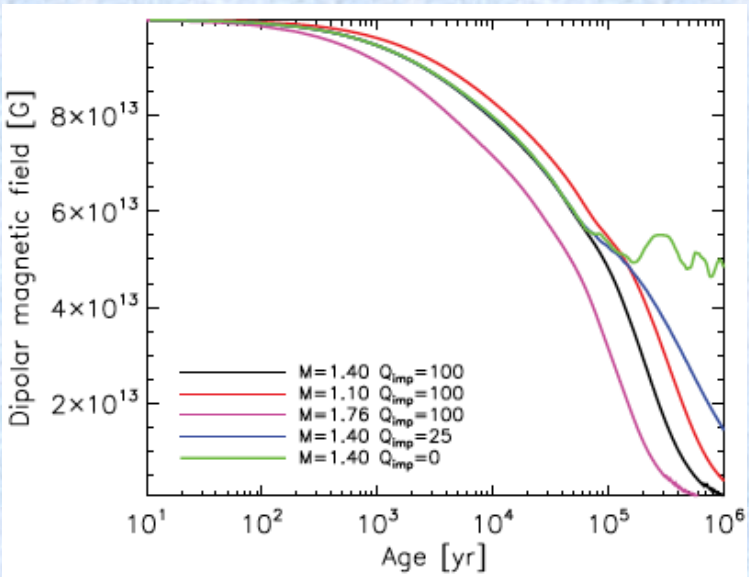
Co-existence of nuclear species → "**impurities**"

→ Static properties (e.g. EoS) similar to ground-state cold catalysed matter

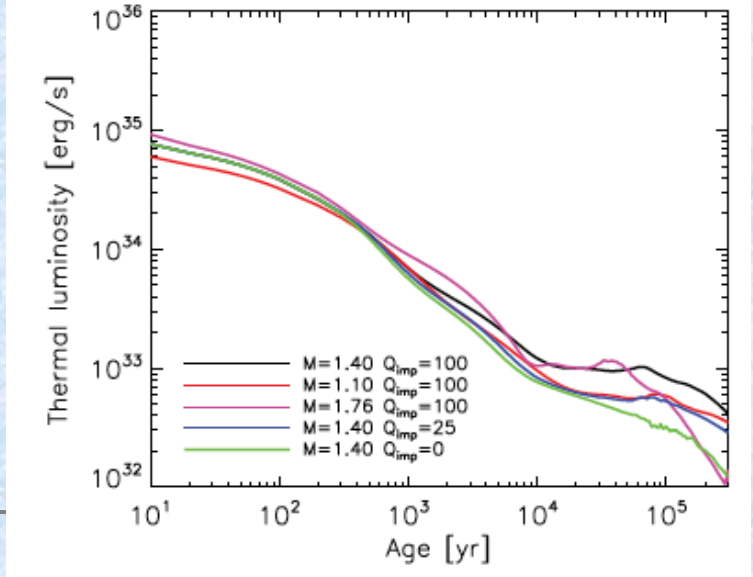
→ Dynamic, magneto-rotational, and transport properties affected by impurities

- ❖ Cooling simulations use ground-state EoS and composition
- ❖ "**Impurity factor**" (free parameter adjusted on observational data)

$$Q_{\rm imp} = \sum_j p(Z^{(j)})(Z^{(j)} - \langle Z \rangle)^2$$



$Q_{\rm imp}$ = 0.1 except above 6 x 10^{13} g cm^{-3}



Viganò et al., MNRAS 434, 123 (2013)

see also Schmidt&Shternin, ASSL 457, 455 (2018) for a review;
Jones, PRL 83, 3589 (1999), MNRAS 321, 167 (2001), PRL 93, 221101 (2001); Pons et al., Nat. Phys. 9, 431 (2013)

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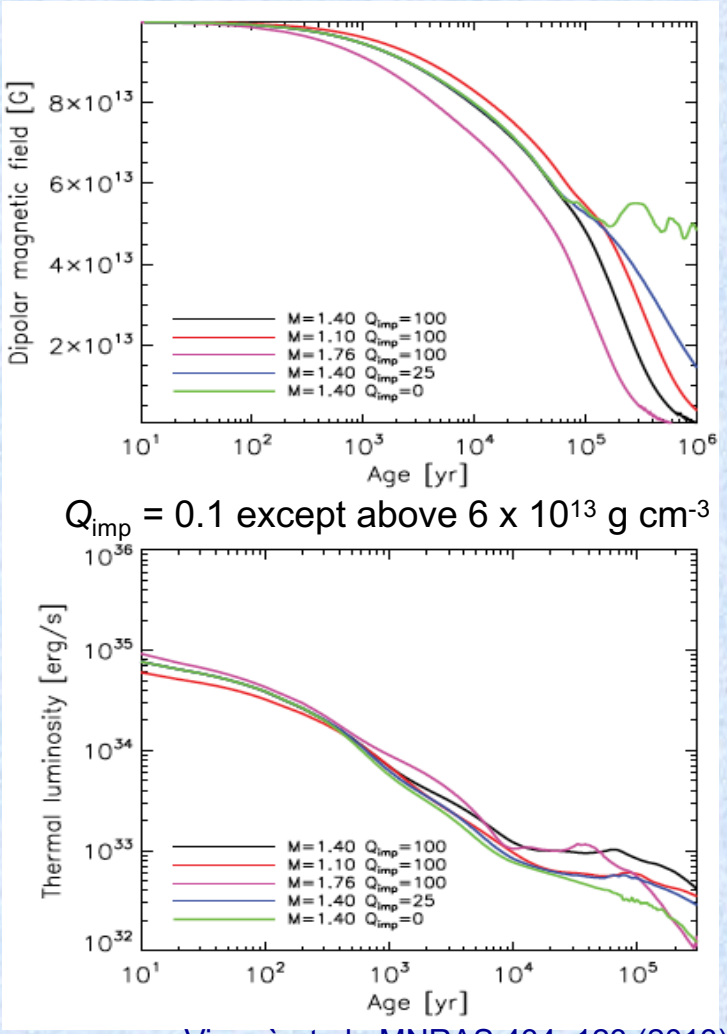

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Viganò et al., MNRAS 434, 123 (2013)

→ **consistent calculation of nuclear distributions and $Q_{\rm imp}$**

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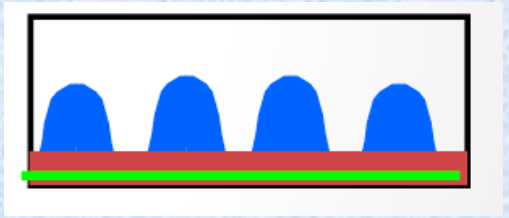
Model : OCP vs MCP



- **OCP** : one nucleus in every Wigner-Seitz cell (all identical)
  → minimization of (Gibbs / free) energy gives favoured *cluster (A,Z)*

$$\mathcal{F}^{\rm OCP} = \mathcal{F}_i(A, Z, n_B, n_p = n_e, T) + \mathcal{F}_e(n_e, T) + \mathcal{F}_n(n_n, T)$$

cluster (mass + translat. (liq) / zero-point vibr. (sol) + Coulomb + corrections) + e gas + n gas (in inner crust)

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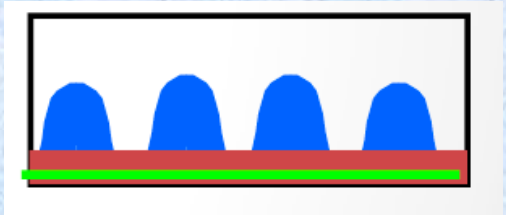

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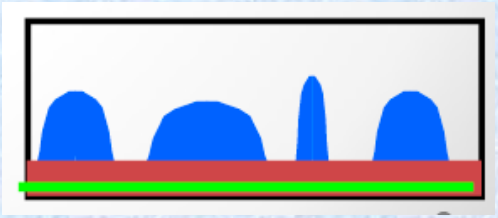
cluster (mass + translat. (liq) / zero-point vibr. (sol) + Coulomb + corrections)
+ e gas + n gas (in inner crust)

- **MCP** : distribution of clusters
  → minimization of (Gibbs / free) energy gives favoured *cluster distribution ($A^{(j)}$,$Z^{(j)}$)*



$$\mathcal{F} = \sum_j n_N^{(j)} F^{(j)} \rightarrow \mathcal{F} = \sum_j p_j \mathcal{F}^{(j)}$$

= 0 in outer crust

$$\mathcal{F}^{(j)} = \mathcal{F}_i^{(j)}(A^{(j)}, Z^{(j)}, n_B^{(j)}, n_p = n_e, T) + \mathcal{F}_e(n_e, T) + \mathcal{F}_n(n_n, T)$$

N.B. : 1. Gas uniform and same in every cell
2. total F is not just the sum of OCP (translational term!)

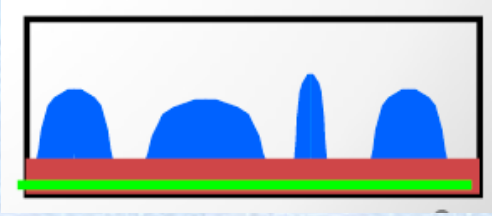
$$F = \sum_j p_j F^{(j)} \neq \sum_j p_j F^{(j),\rm OCP}$$

see e.g. Gulminelli&Raduta, PRC 92, 055803 (2015);
for a general review on approaches, see e.g. Oertel et al., Rev. Mod. Phys. 89, 015007 (2017);
Burgio&Fantina, ASSL 457, 255 (2018)

Model : MCP

- **MCP** : distribution of clusters in the liquid phase

Minimization of energy density under charge and baryon number constraints:



$$p_j \propto \exp\left\{-\frac{1}{k_B T}\left[F_i^{(j),\mathrm{OCP}} + \delta\Omega^{(j)} - \mu_n N^{(j)} - \mu_p Z^{(j)}\right]\right\}$$

→ to determine cluster probabilities p_j one needs :

- Chemical potentials μ_p, μ_n (with β-equil. condition)
- Free energies $F^{(j)} = F_i^{(j)} + F_e^{(j)}$ (+ $F_n^{(j)}$ in the inner crust). F_i and F_n depend on the nuclear model adopted;
- Rearrangement term $R^{(j)}$: $\delta\Omega^{(j)} = k_B T(\ln V^{(j)} + 1) + \mathcal{R}^{(j)}$

N.B. : Rearrangement term R because of dependence of F_i on local density due to charge conservation. Often neglected! Small but needed to ensure thermo consistency !

$$n_e = \sum_j n_N^{(j)} Z^{(j)}$$

see Grams et al., PRC 97, 035807 (2018); Fantina et al., A&A 633, A149 (2020); Carreau et al., A&A 640, A77 (2020)

Thermo conditions for crystallization

From the thermodynamic point of view, T_m is determined from Gibbs condition of phase equilibrium, i.e. a set of coupled equations for the *m* component of the MCP:

$$\frac{\partial F_{\mathrm{sol}}^{\mathrm{MCP}}}{\partial p_j}(p_{1,\mathrm{sol}},\ldots,p_{m,\mathrm{sol}}) = \frac{\partial F_{\mathrm{liq}}^{\mathrm{MCP}}}{\partial p_j}(p_{1,\mathrm{liq}},\ldots,p_{m,\mathrm{liq}})$$

$$F_{\mathrm{sol}}^{\mathrm{MCP}} = F_{\mathrm{liq}}^{\mathrm{MCP}} + \nabla_{\mathbf{p}} F_{\mathrm{liq}}^{\mathrm{MCP}} \cdot (\mathbf{p}_{\mathrm{sol}} - \mathbf{p}_{\mathrm{liq}}) \qquad \nabla_{\mathbf{p}} \to \partial/\partial p_j$$

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$$F_{\rm sol}^{\rm MCP} = F_{\rm liq}^{\rm MCP} + \nabla_{\mathbf{p}} F_{\rm liq}^{\rm MCP}\cdot(\mathbf{p}_{\rm sol}-\mathbf{p}_{\rm liq}) \qquad \nabla_{\mathbf{p}} \to \partial/\partial p_j$$

but: full phase equilibrium eqs. should be solved → numerically costly
→ we assume the crystallization condition of OCP :

$$f_{\rm liq}^{\rm OCP} = f_{\rm sol}^{\rm OCP}$$

or, at constant *P* :

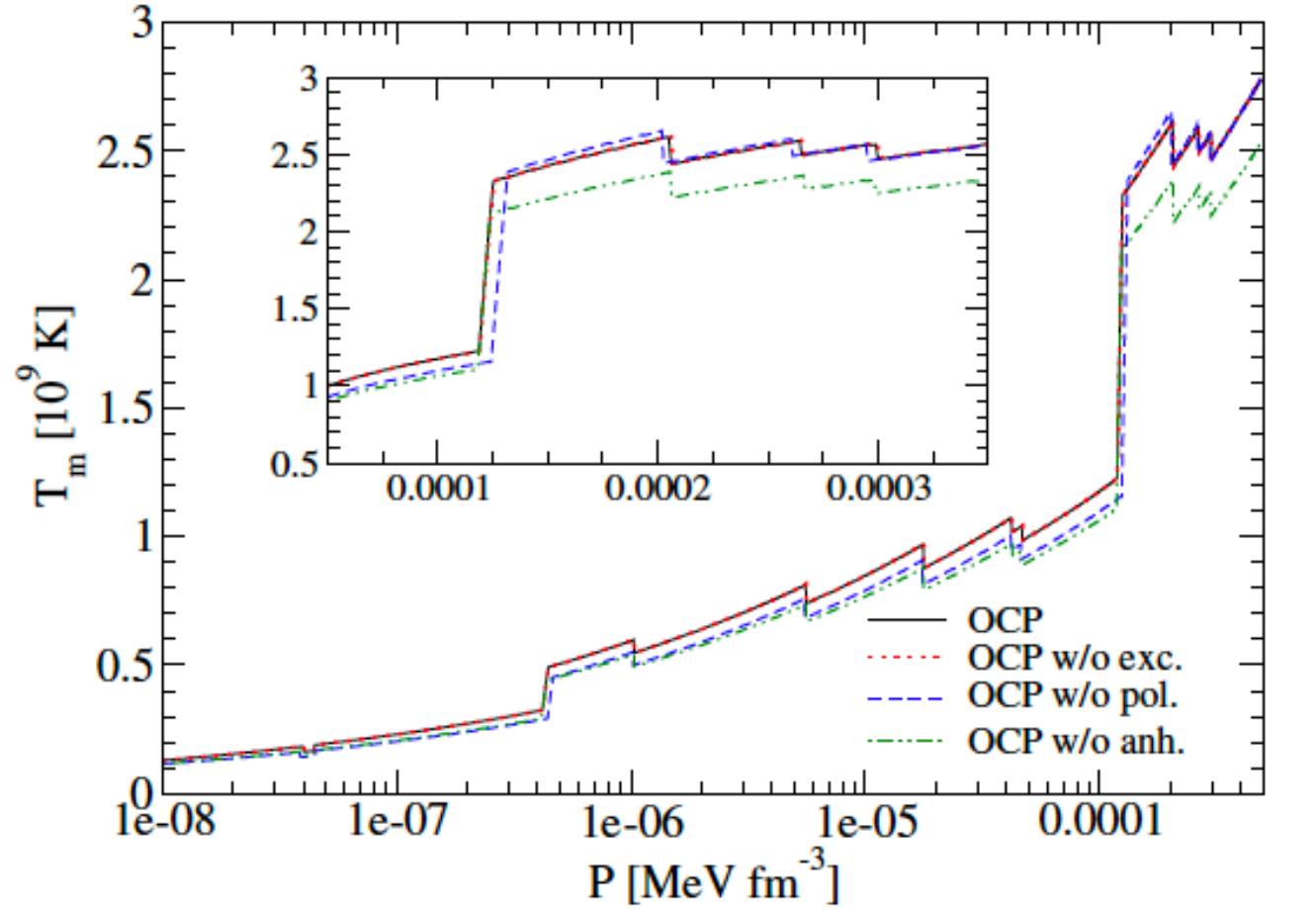
$$g_{\rm liq}^{\rm OCP} = g_{\rm sol}^{\rm OCP}$$

- ✓ Simpler condition, but computationally faster → decrease *T* until T_m is found
- ✓ At T_m : consistent calculation of distribution in (liquid) MCP → $Q_{\rm imp}$

$$Q_{\rm imp} = \sum_j p(Z^{(j)})(Z^{(j)} - \langle Z\rangle)^2$$

N.B.: if composition frozen at different temperature T_f
→ $Q_{\rm imp}$ has to be calculated at T_f

Outer crust: T_m for OCP



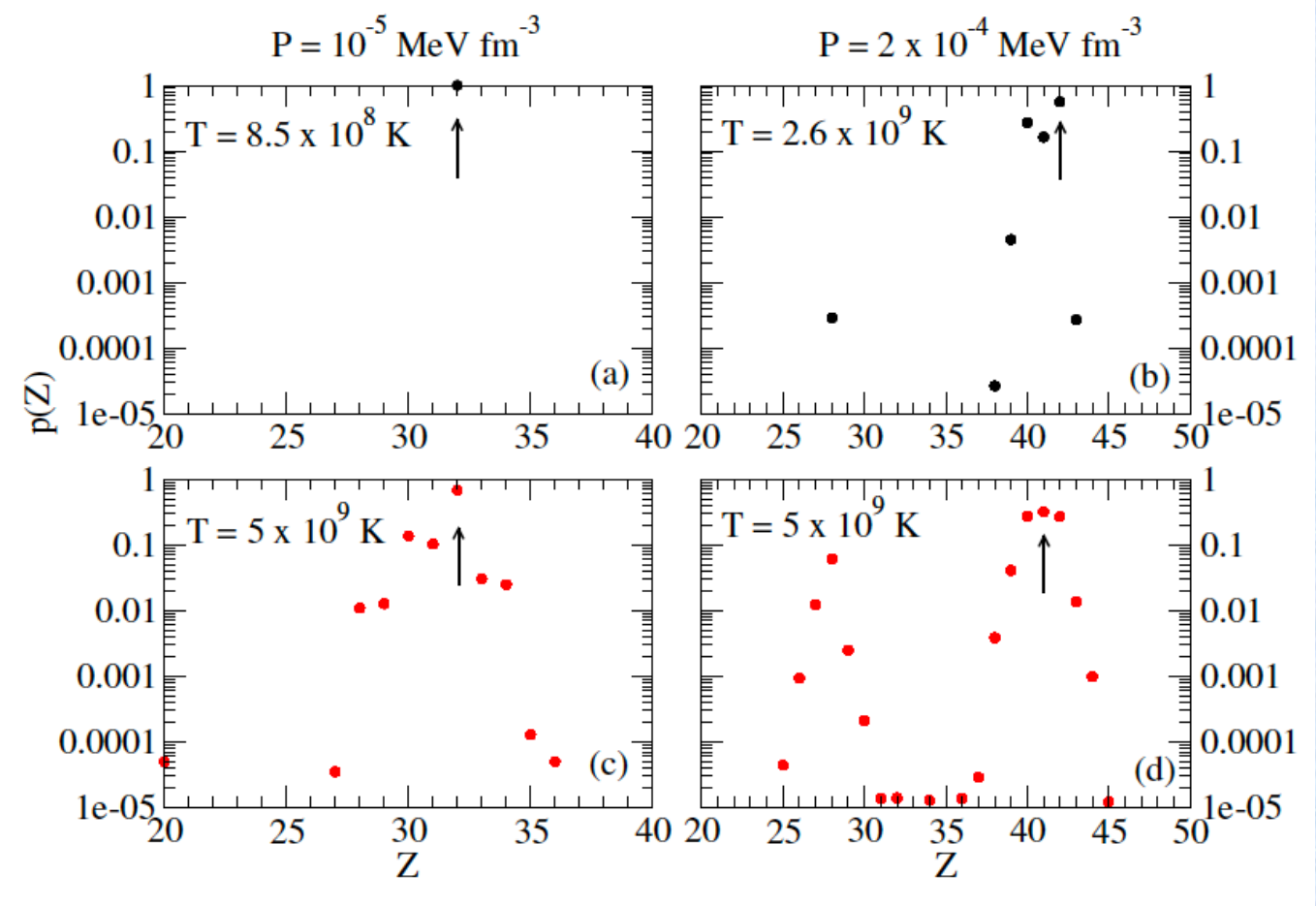
Fantina et al., A&A 633, A149 (2020)

10^8 < T_m < few 10^9 K

Small energy corrections have negligible impact on energetics but can affect T_m

Exp (AME2016) + HFB-24 masses (Goriely et al, PRC 88, 024308 (2013))

Outer crust : OCP vs MCP composition



Fantina et al., A&A 633, A149 (2020)

Exp + HFB-24 masses

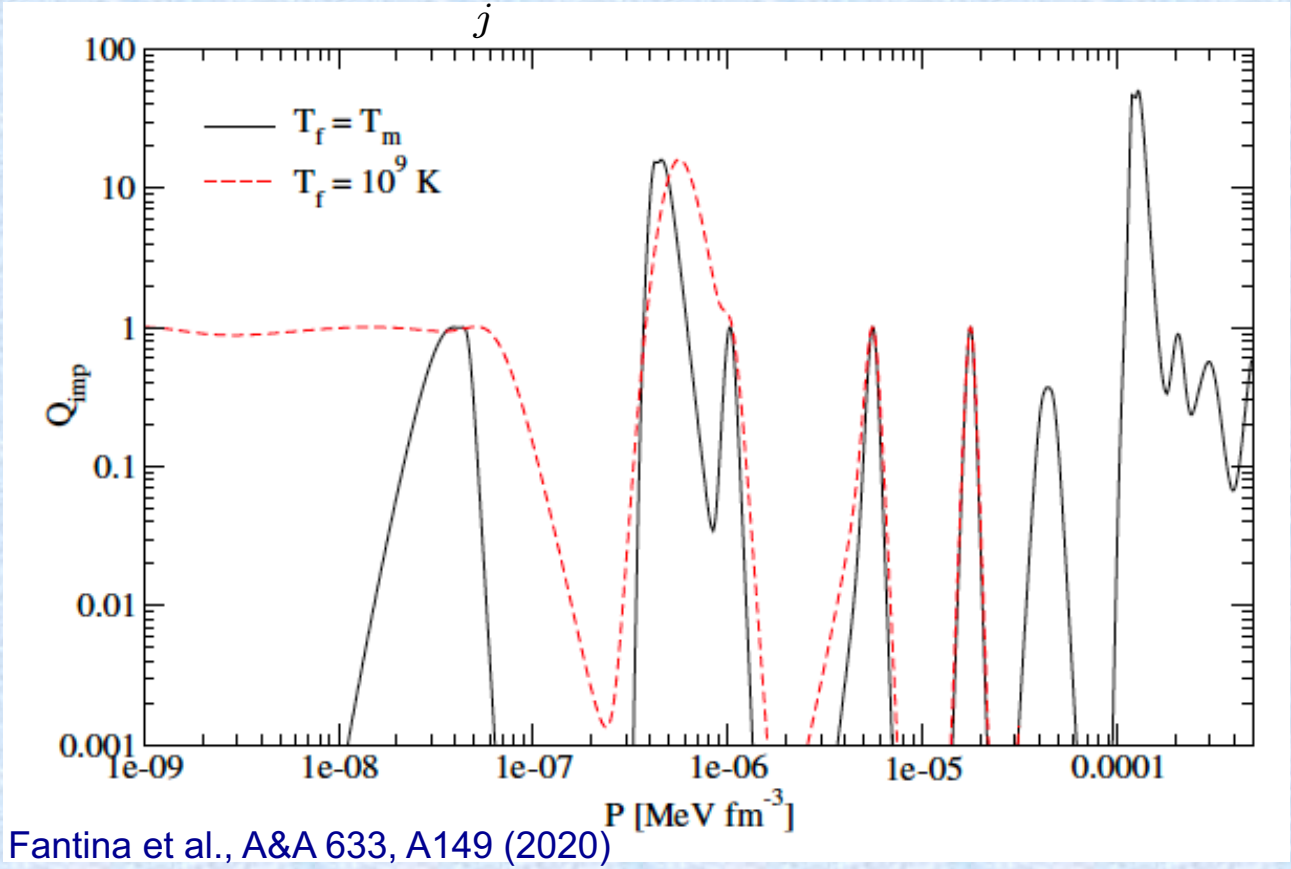
→ OCP approx. very good at lower *P, T*

→ linear mixing rule very good approx. in liquid phase

→ bimodal distribution at higher *T*
→ higher Q_{imp}

Outer crust: impurity parameter

$$Q_{\rm imp} = \sum_j p(Z^{(j)})(Z^{(j)} - \langle Z \rangle)^2$$

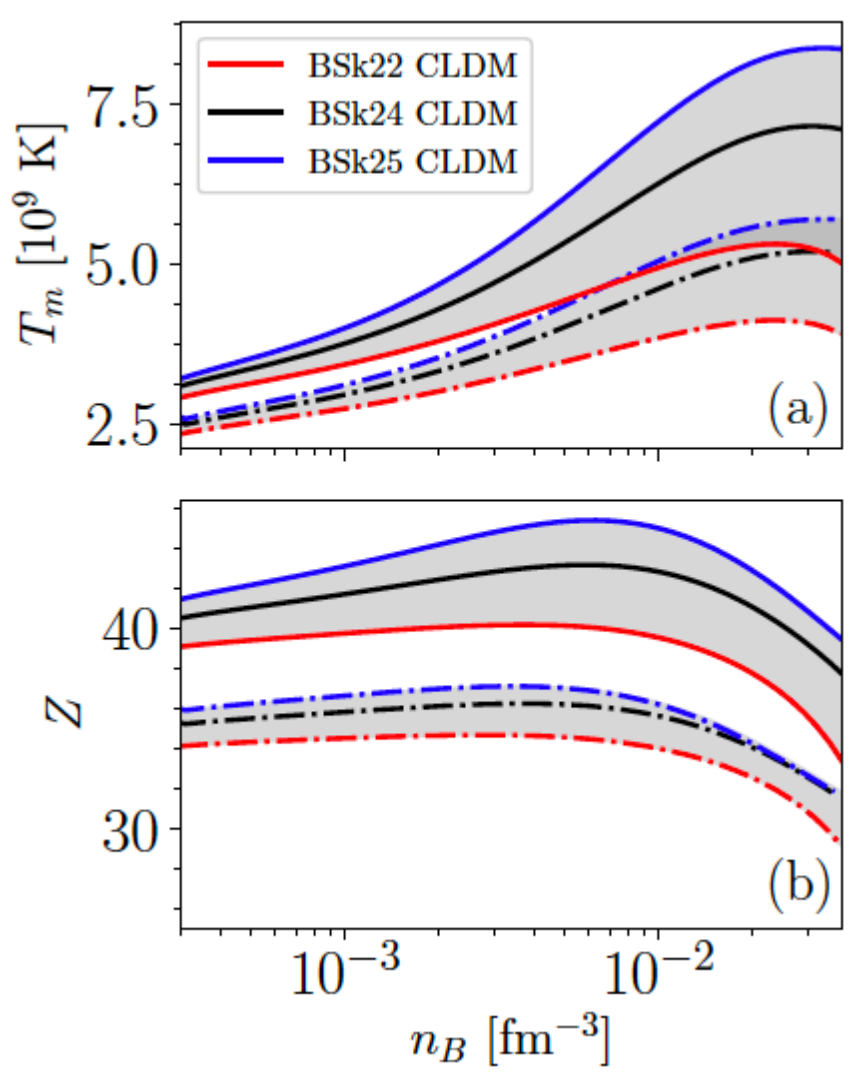


Fantina et al., A&A 633, A149 (2020)

- Small Q_{imp} → distribution peaked → OCP good approx.
- Variation of Q_{imp} → alternation of pure (conductive) and impure (resistive) layers

N.B. In various work Q_{imp} was fixed (constant)

Inner crust: T_m and Z for OCP



Band: uncertainties on the EoS
(BSk22-25 differ for their symmetry energy)

Solid lines: CLD results with surface parameters optimized on ETF calculations

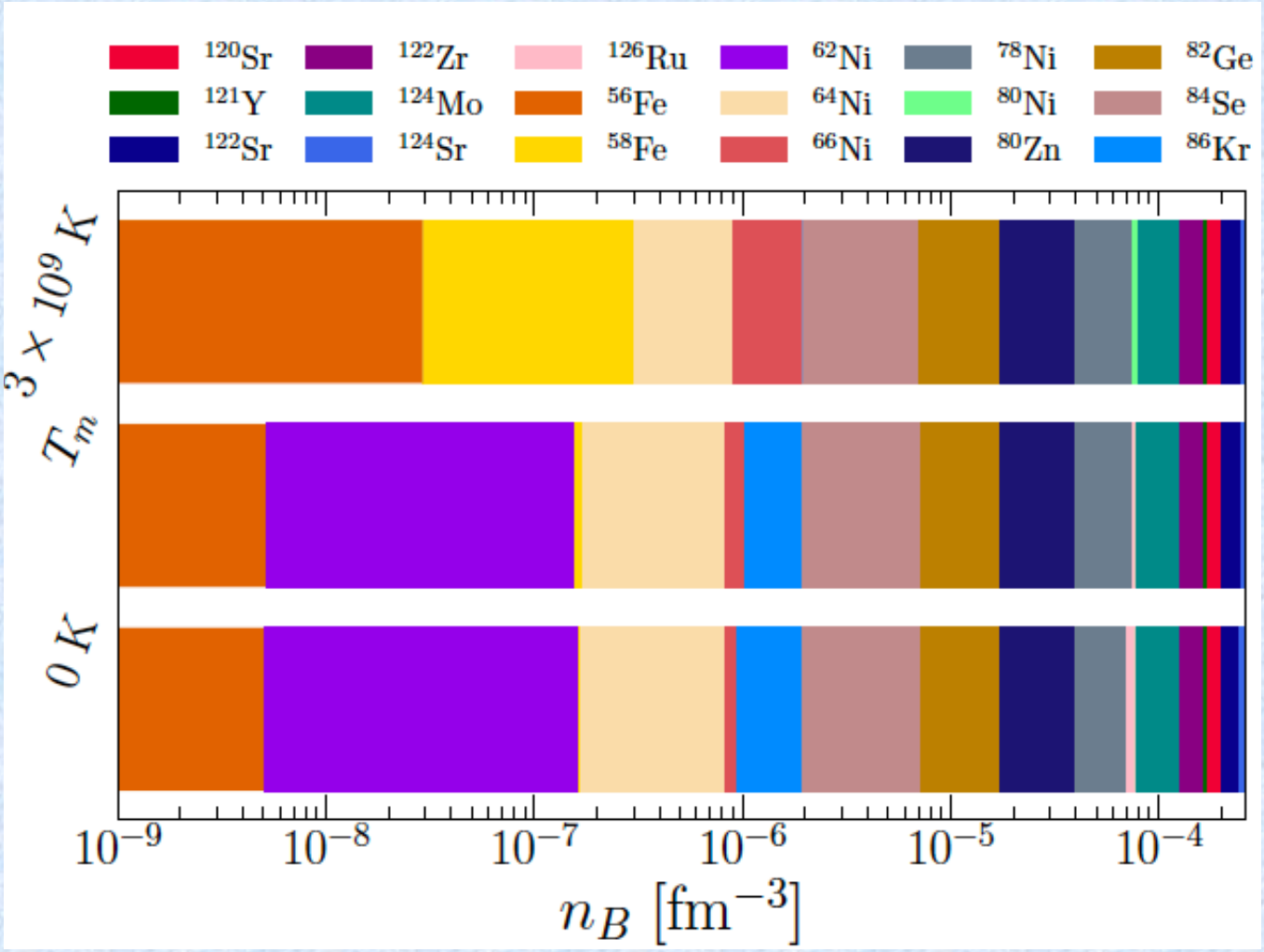
Dot-dashed: CLD results with surface parameters optimized on spherical nuclei

→ uncertainties on nuclear EoS and surface energy not negligible

→ uncertainties on surface energy more important than uncertainties on EoS

Carreau et al., A&A 635, A84 (2020)

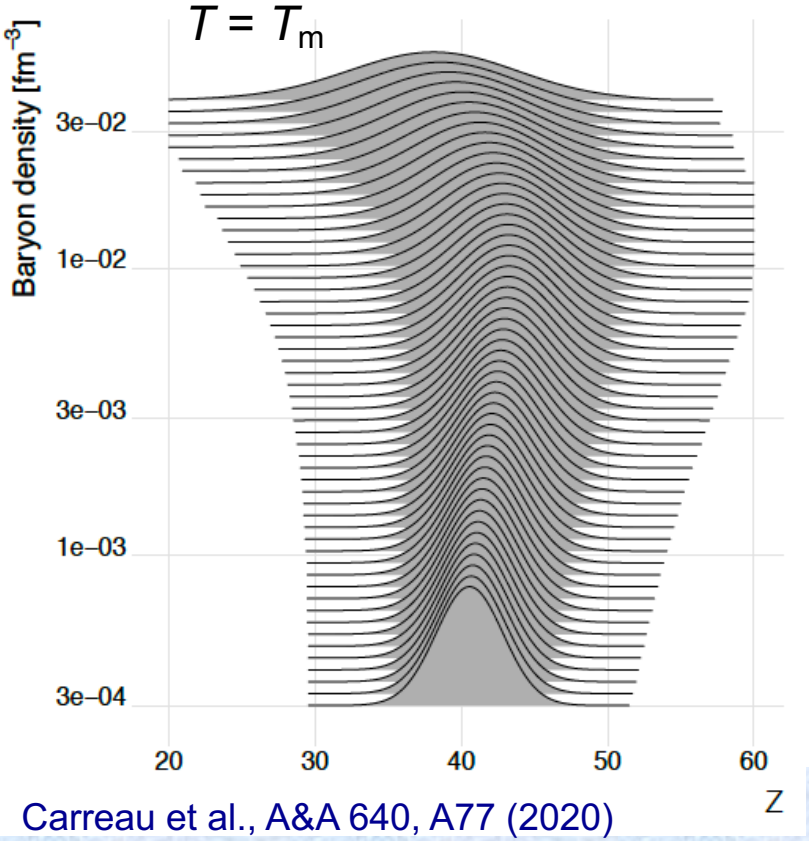
Crust: composition at finite T



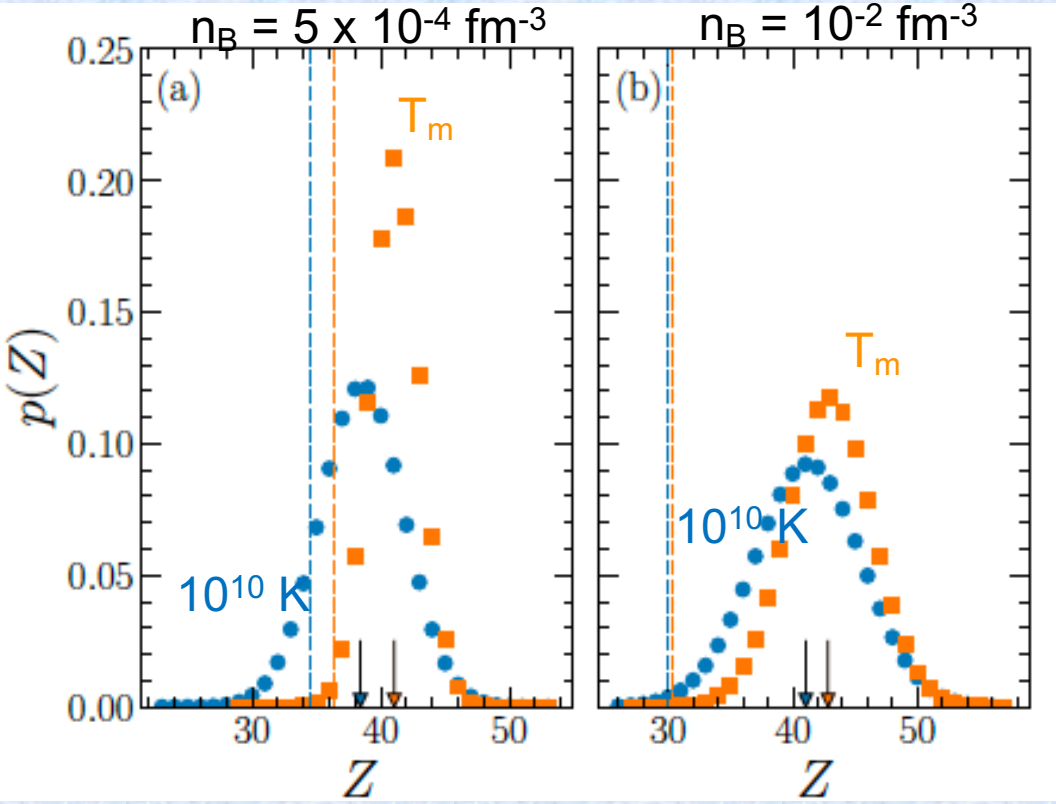
T. Carreau, PhD thesis (2020)

➡ If composition frozen at $T > T_m$ → different from ground state !

Inner crust : OCP vs MCP composition

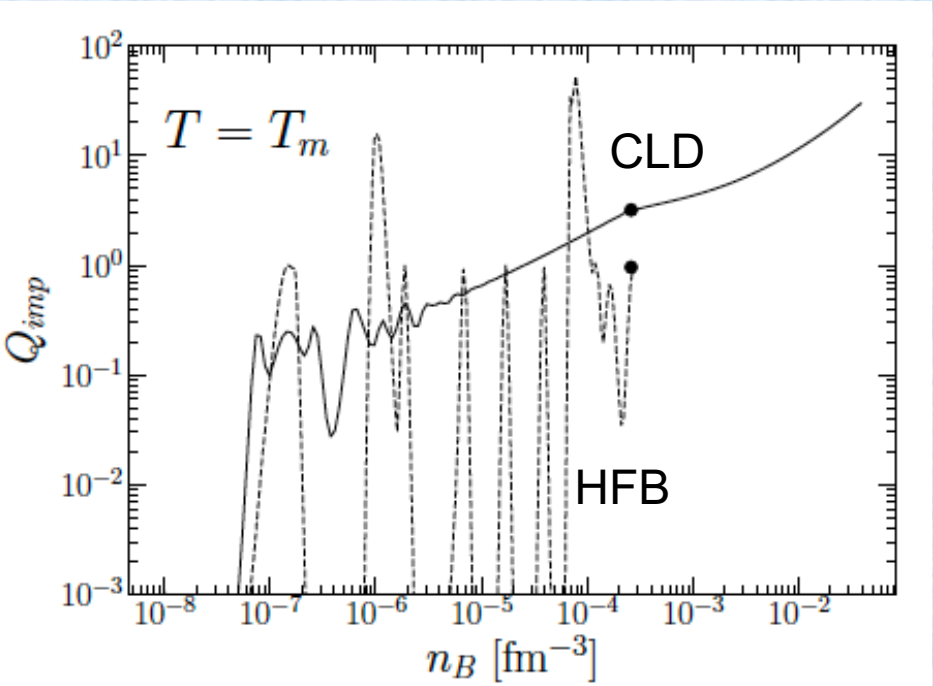


Carreau et al., A&A 640, A77 (2020)



→ MCP average close to OCP also in the inner crust
→ linear mixing rule good approximation in liquid phase
→ distribution larger at larger density (and temperature)
→ Rearrangement term significant and needed for thermodynamic consistency

Inner crust: impurity parameter



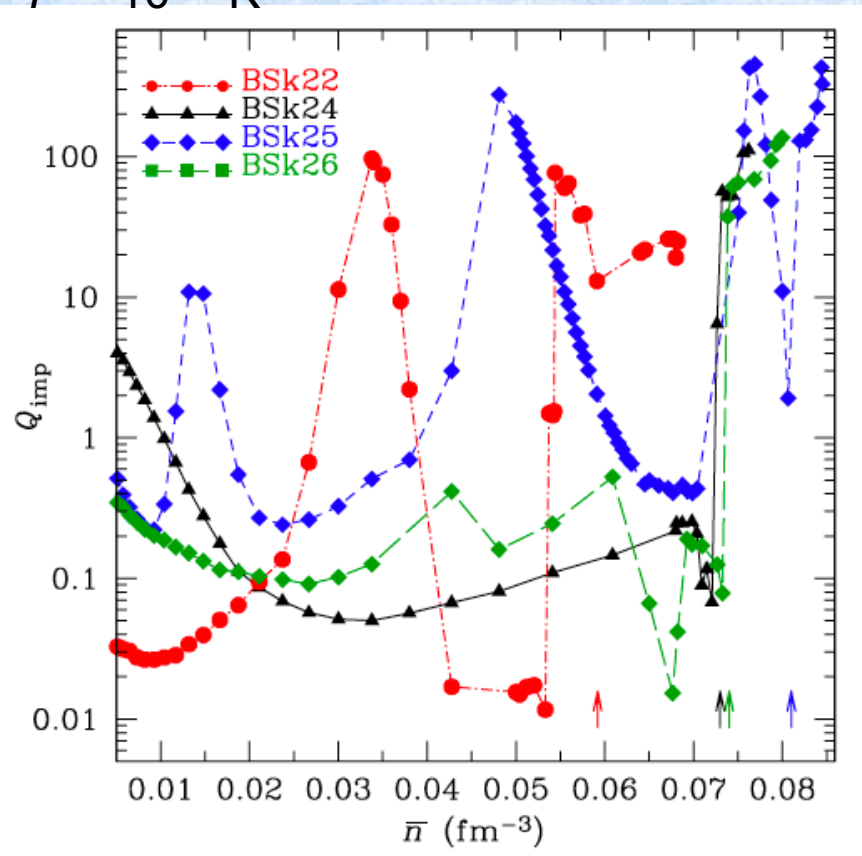
Carreau et al., A&A 640, A77 (2020)

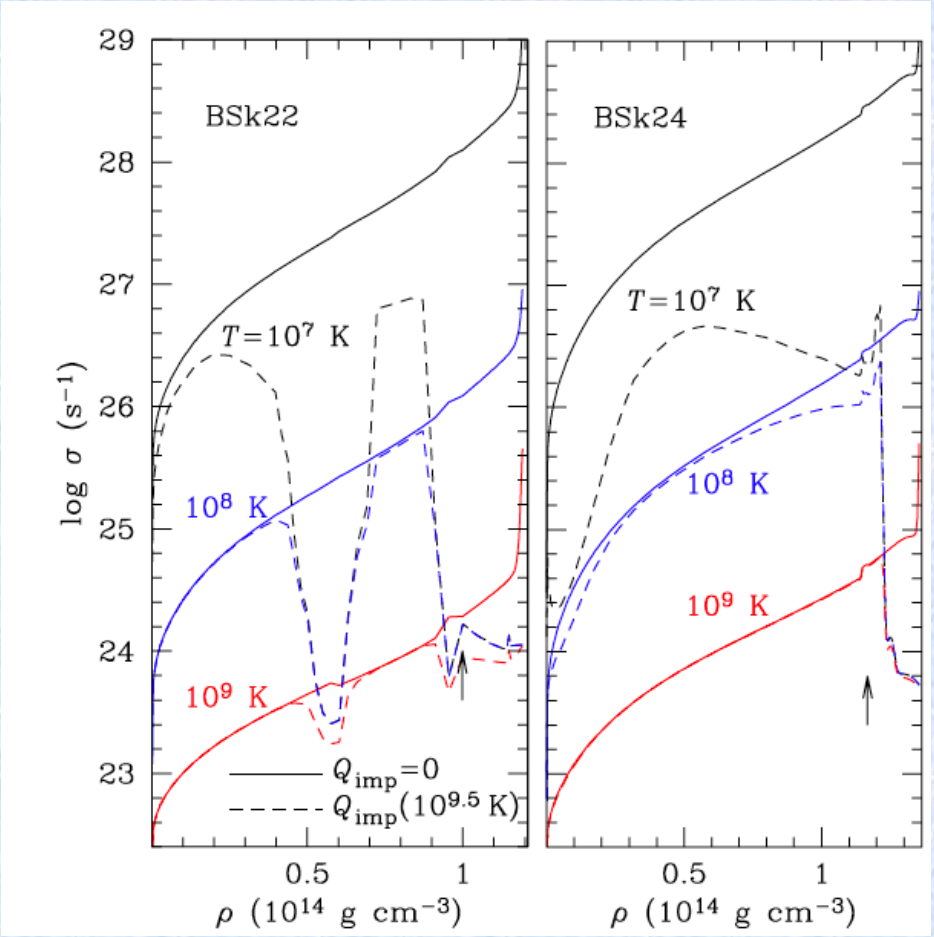
- Q_{imp} larger in inner crust
  → OCP approx. less reliable
- Fluctuations of full micro approach smoothed in CLD approach

Inner crust: impurity parameter

Different approach: inclusion of shell effects but approx. calculation of abundances
→ impact on electrical conductivity

$T = 10^{9.5}$ K





Potekhin & Chabrier, A&A 645, A102 (2021)

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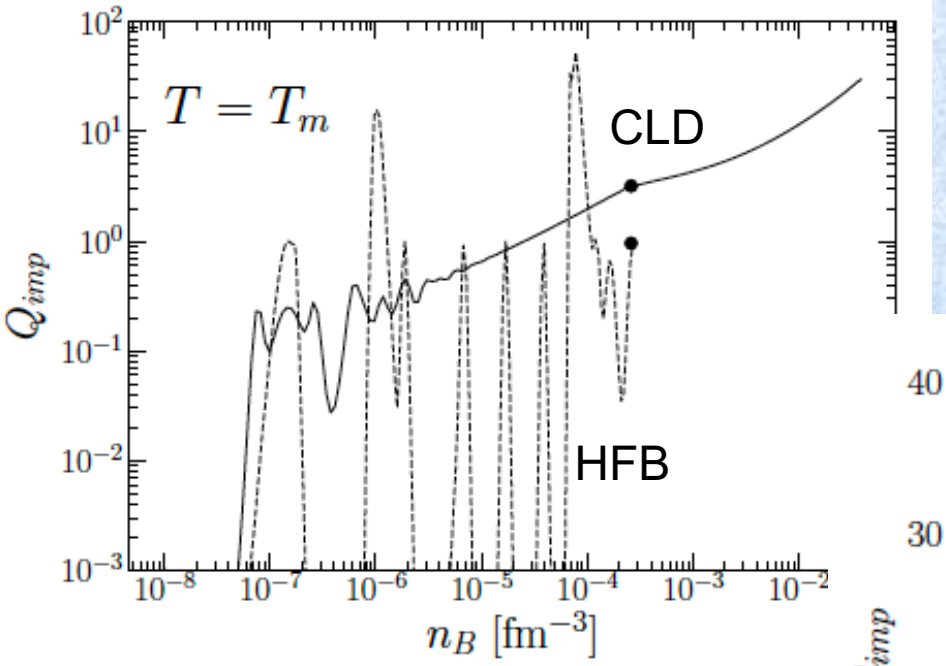

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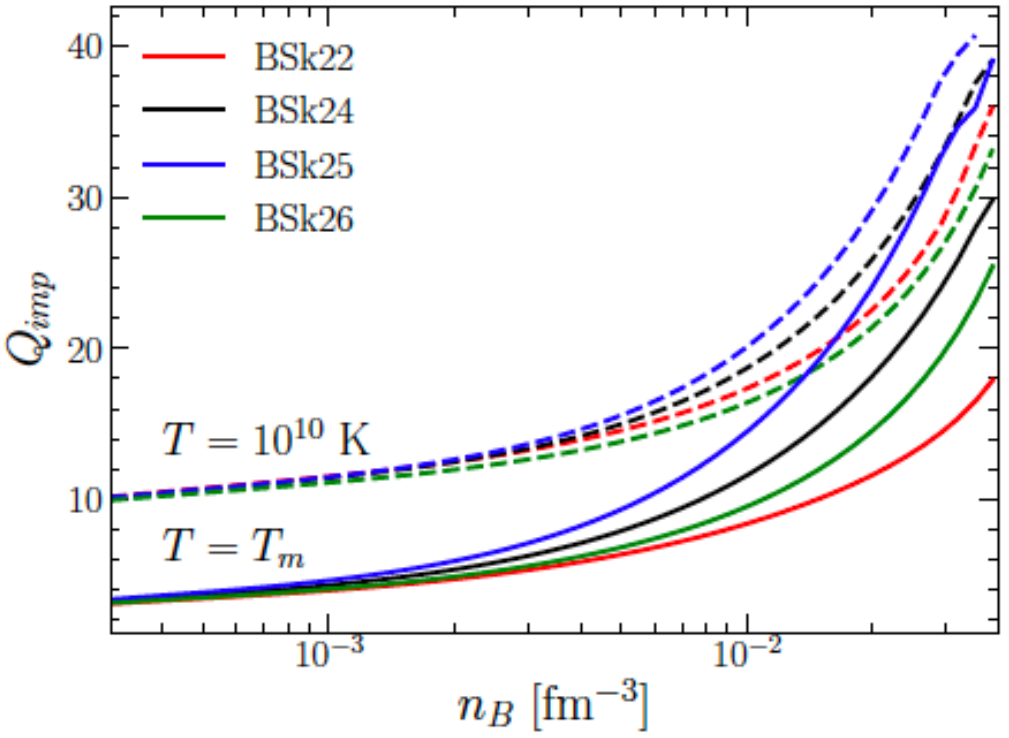

- Larger spread at higher density
- Larger Q_{imp} at higher T

Conclusions & outlooks

- At crystallization composition (and EoS) similar to $T = 0$ one
  → static properties not much affected
- Linear mixing rule good approximation in the liquid phase
- Importance of uncertainties on surface terms at extreme isospin
- Consistent calculation of **distribution** in equilibrium MCP & of **impurity parameter**
  → variation of Q_{imp} → alternation of pure (conductive) and impure (resistive) layers
  → possible impact on transport properties
- Tables of T_m and Q_{imp} available at CDS* (CompOSE ?) → applicable in simulations

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- Impurity factor : can be increased if “pasta” phase at higher density
  (work in progress)
- Transport coefficients/properties → simulations

* http://cdsarc.u-strasbg.fr/viz-bin/cat/J/A+A/633/A149
http://cdsarc.u-strasbg.fr/viz-bin/cat/J/A+A/640/A77


Thank you