Ab initio superfluid properties for neutron star modelling

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CompOSE2021 Workshop
PHAROS WG1+WG2 Meeting
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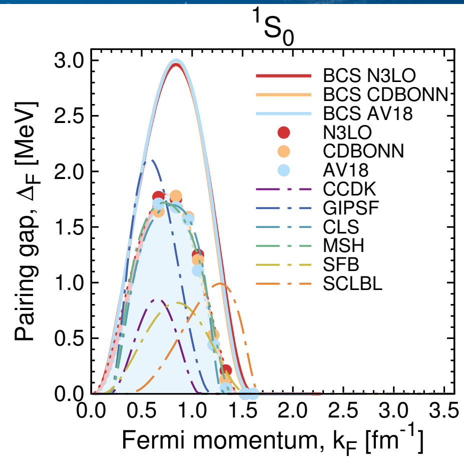
IEEC/Online

Departament de Física Quàntica i Astrofísica

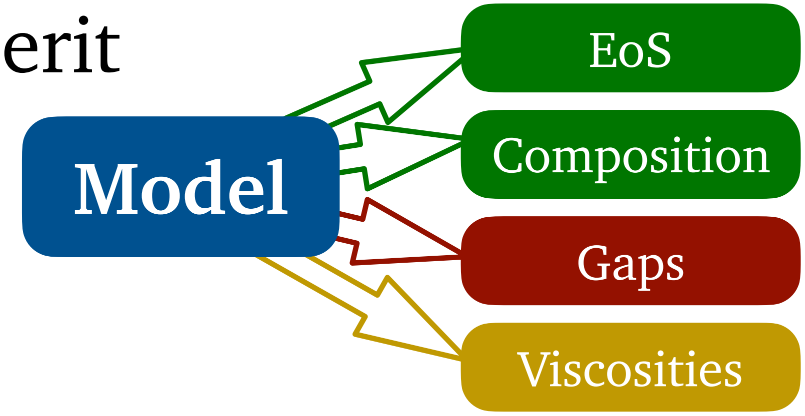
@HadNucAtUB

Wish list for COMPOSE

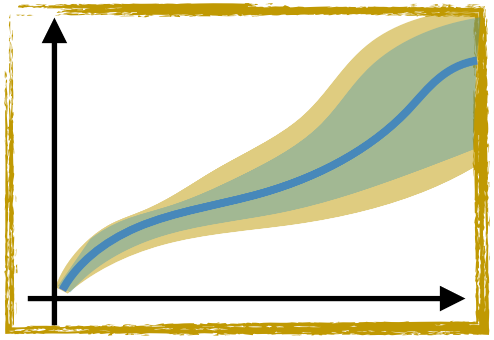
1) Pairing gaps ($^1S_0$ & $^3PF_2$ ideally)



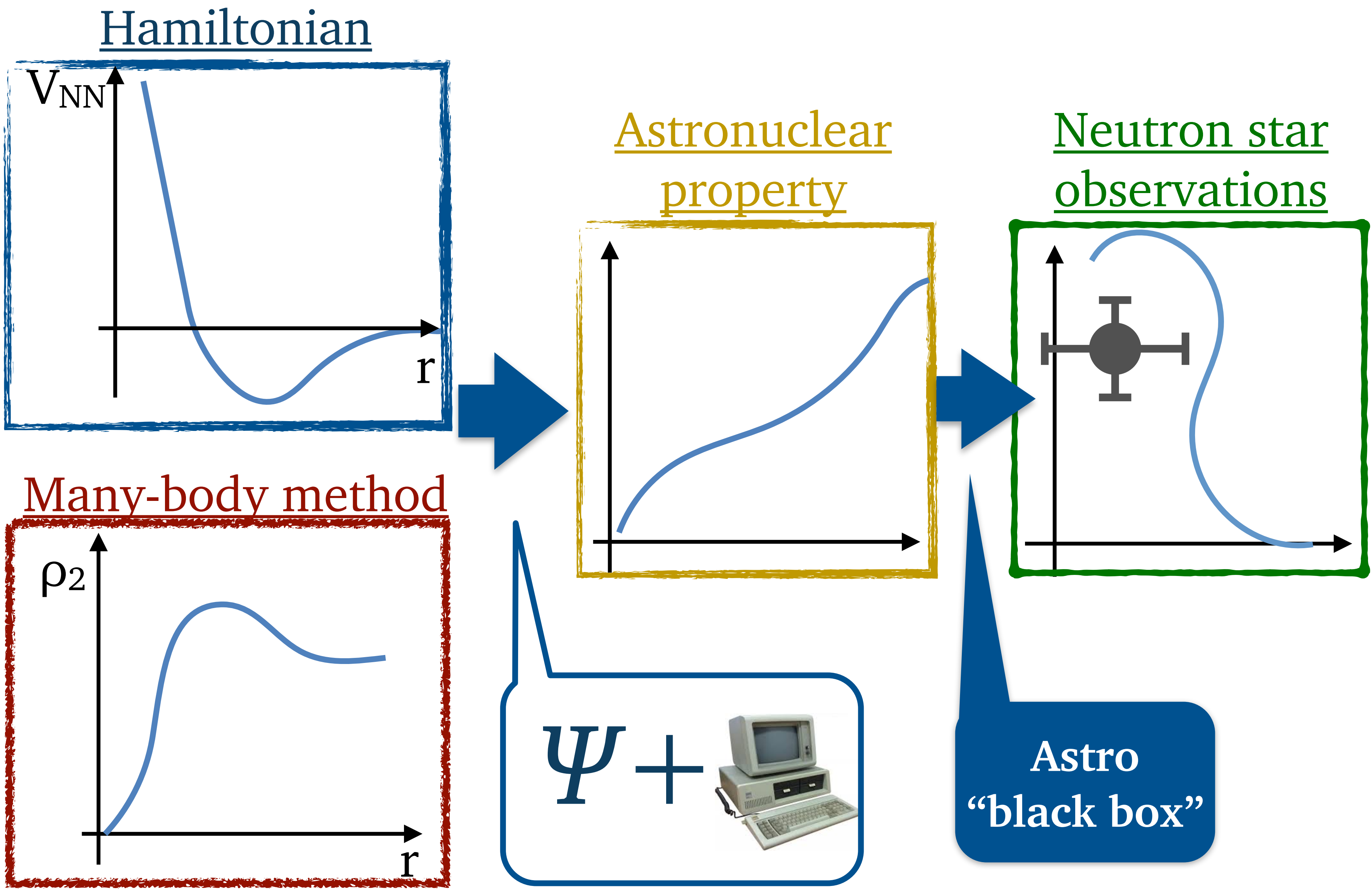
2) "Consistency" figure of merit



3) Uncertainties in predictions?



Nuclear predictions 19xx style



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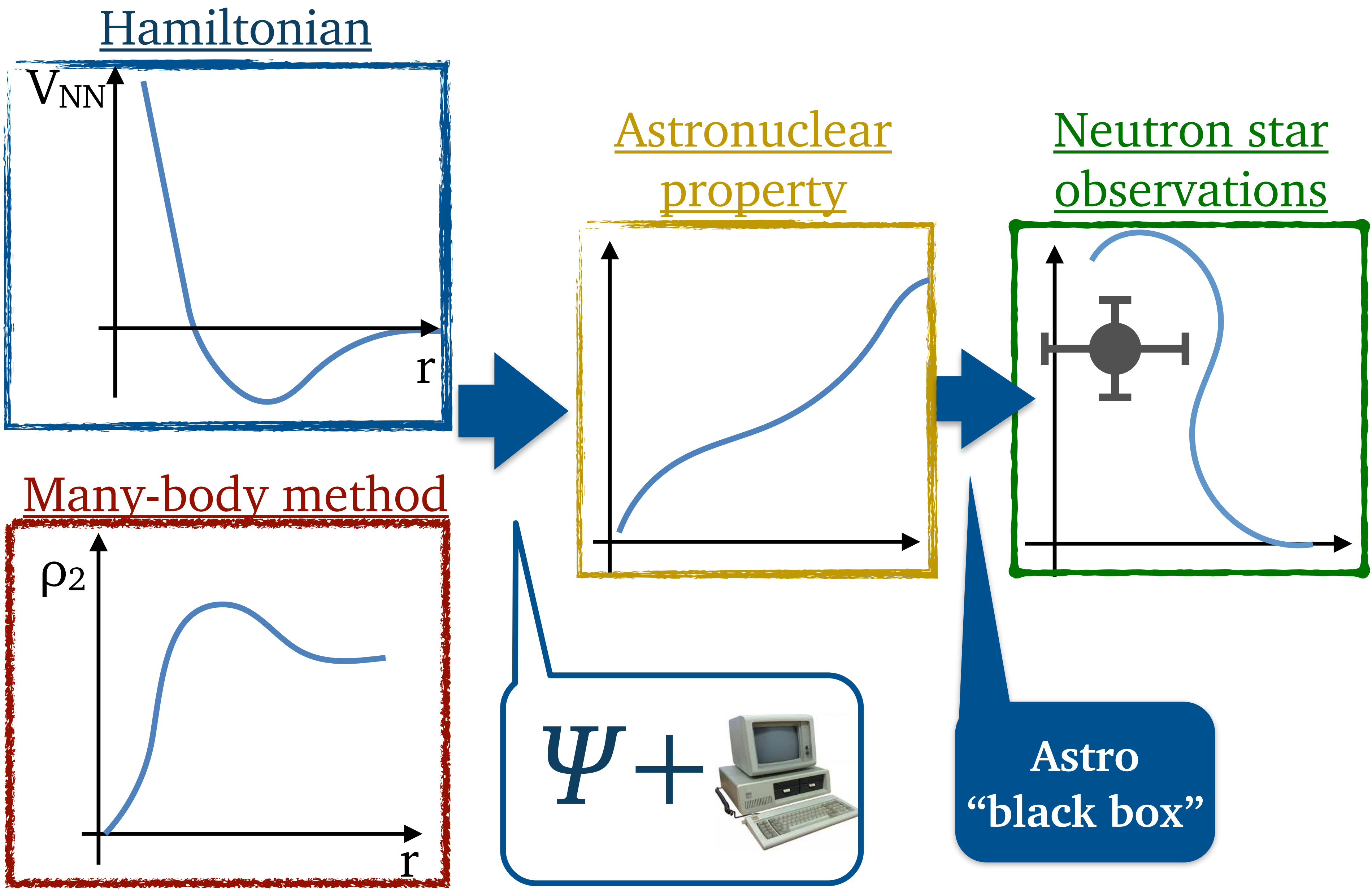

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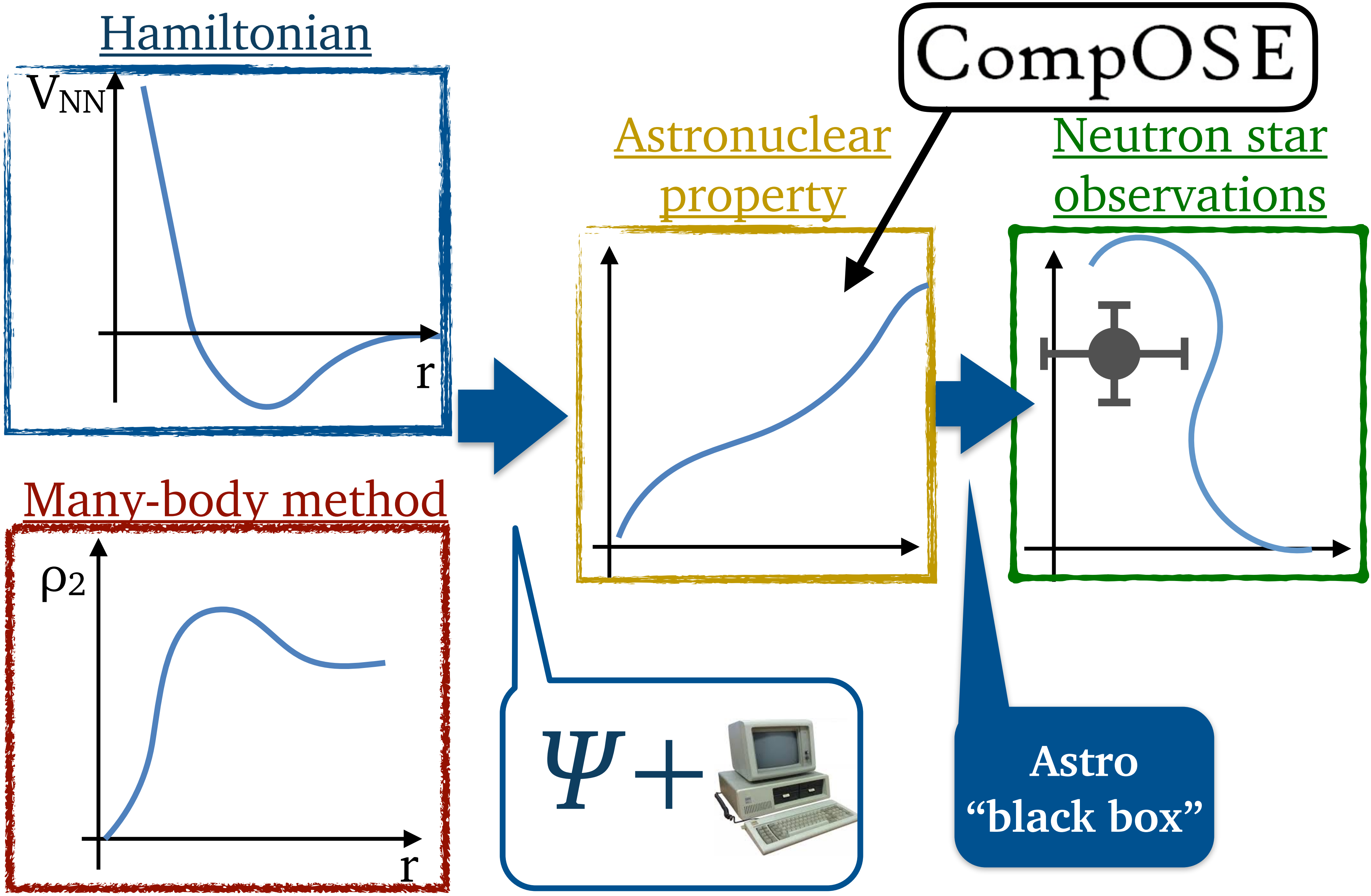

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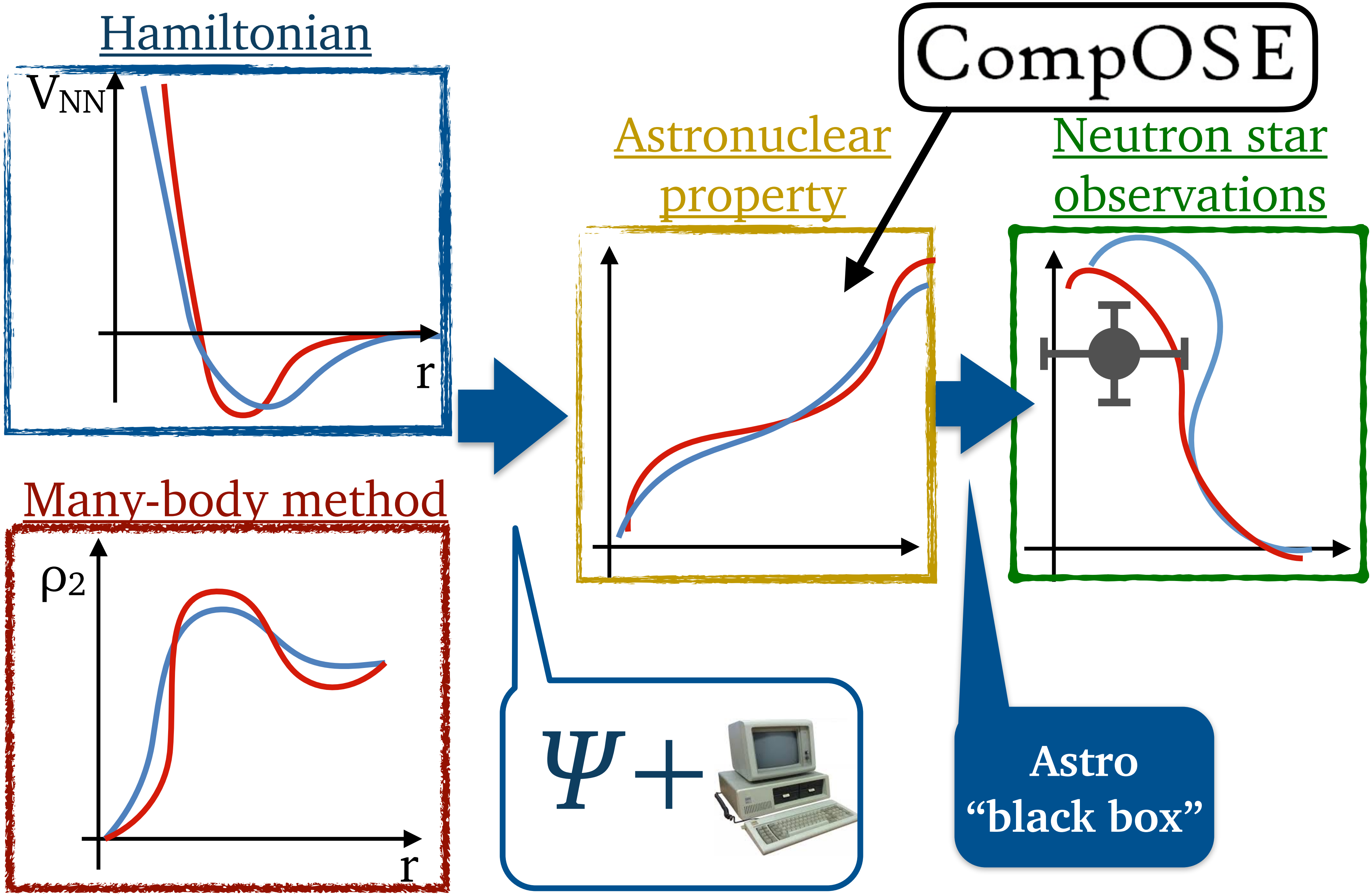

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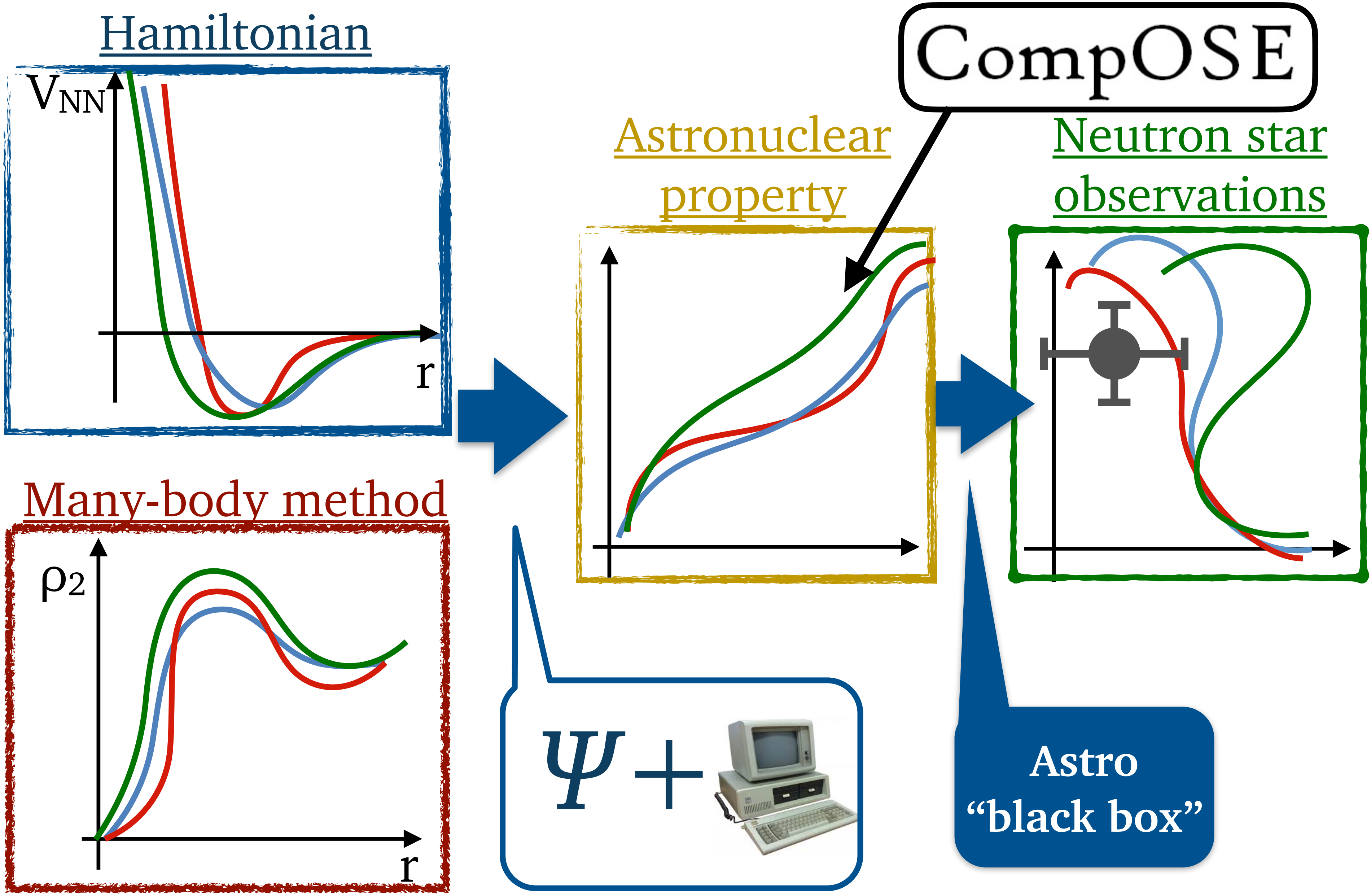
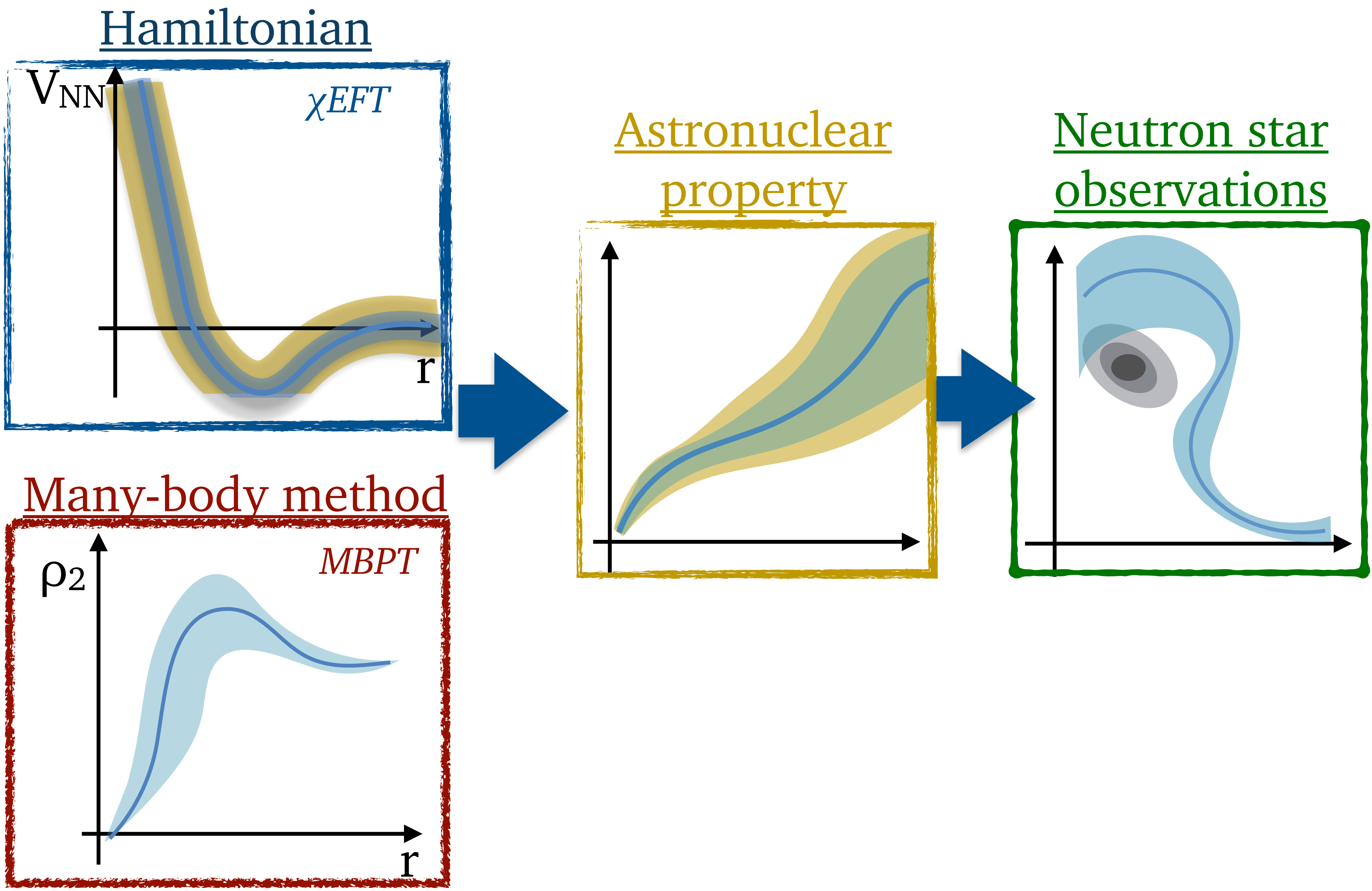
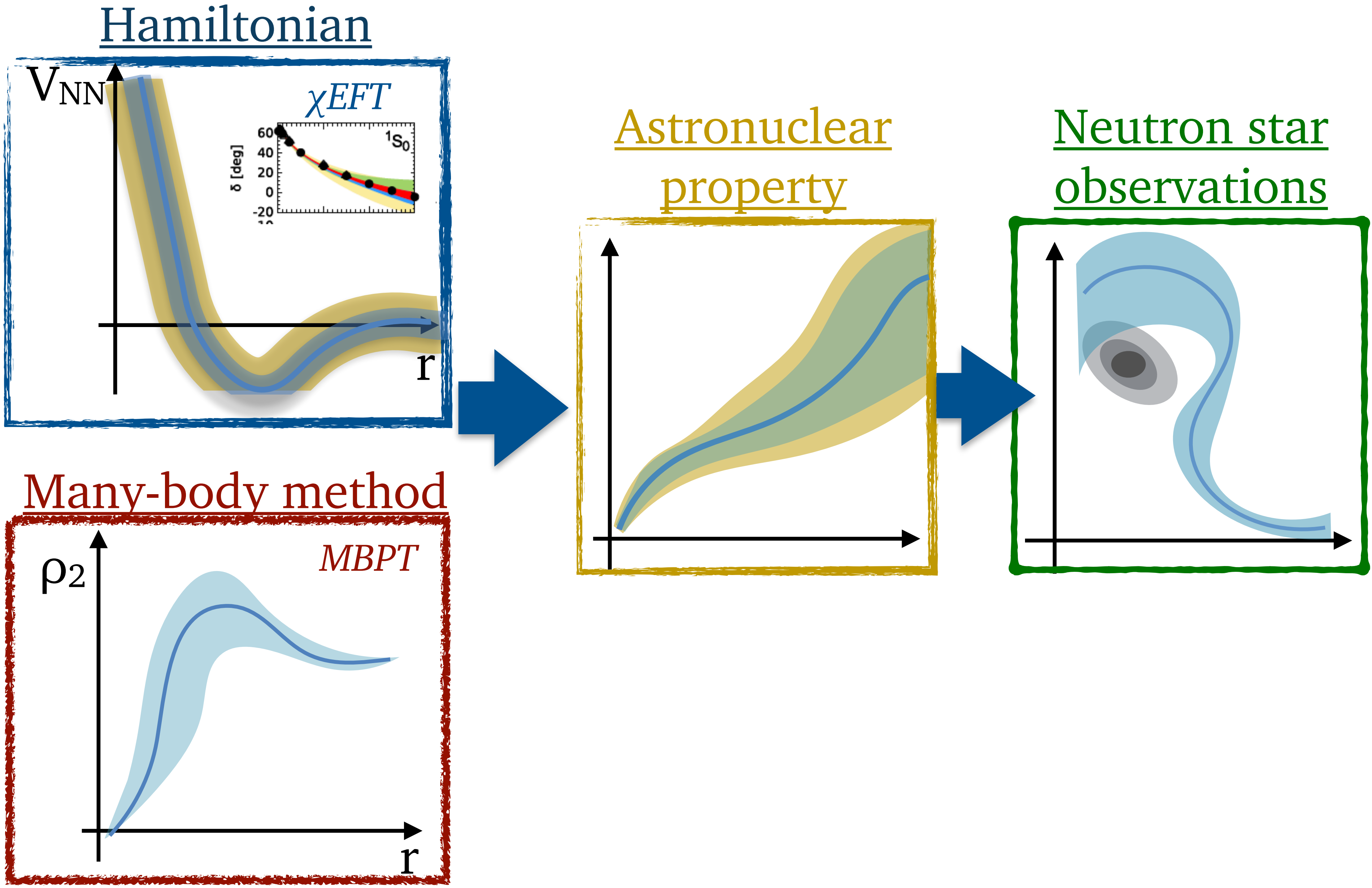


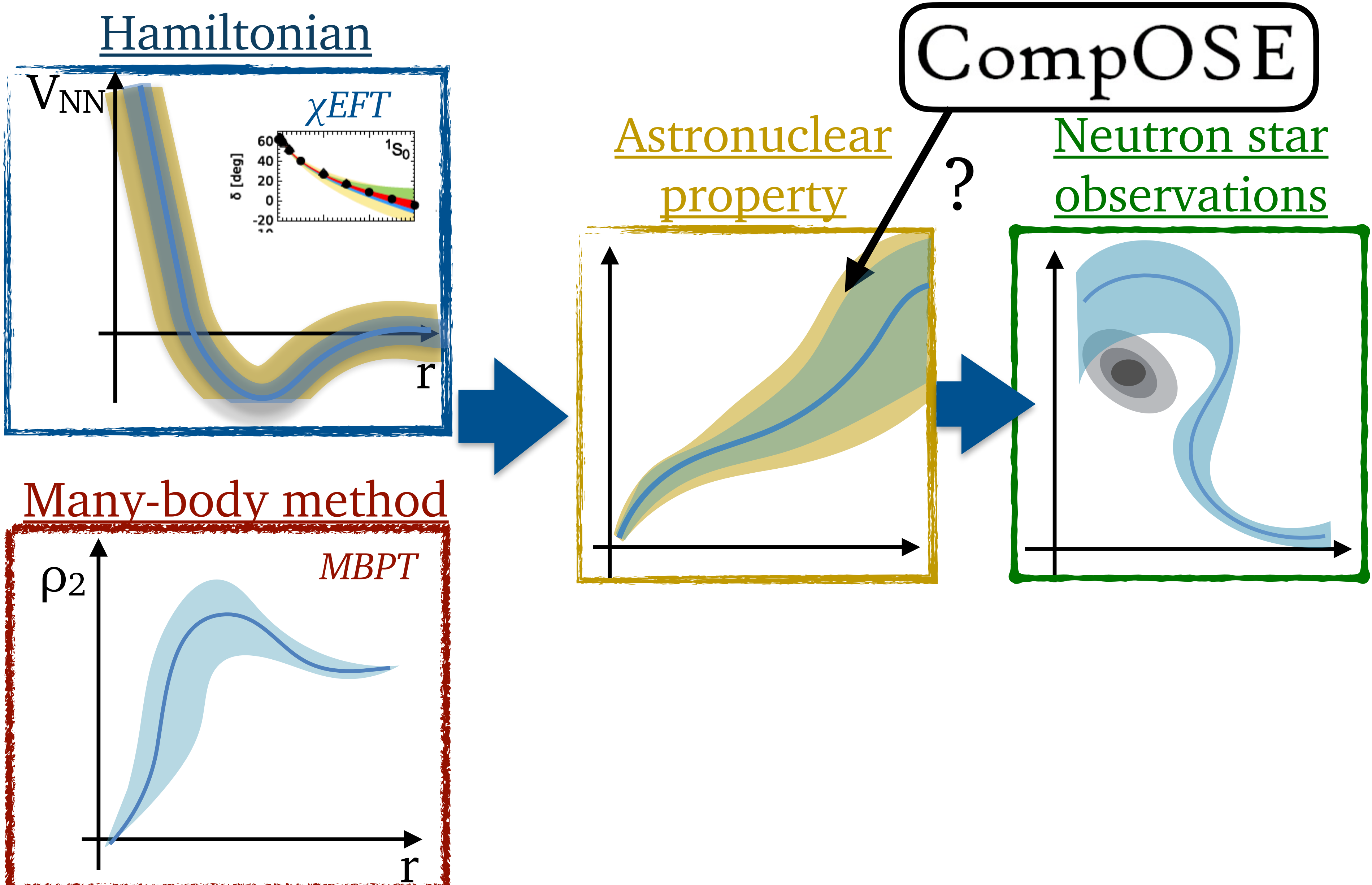
Nuclear error quantification



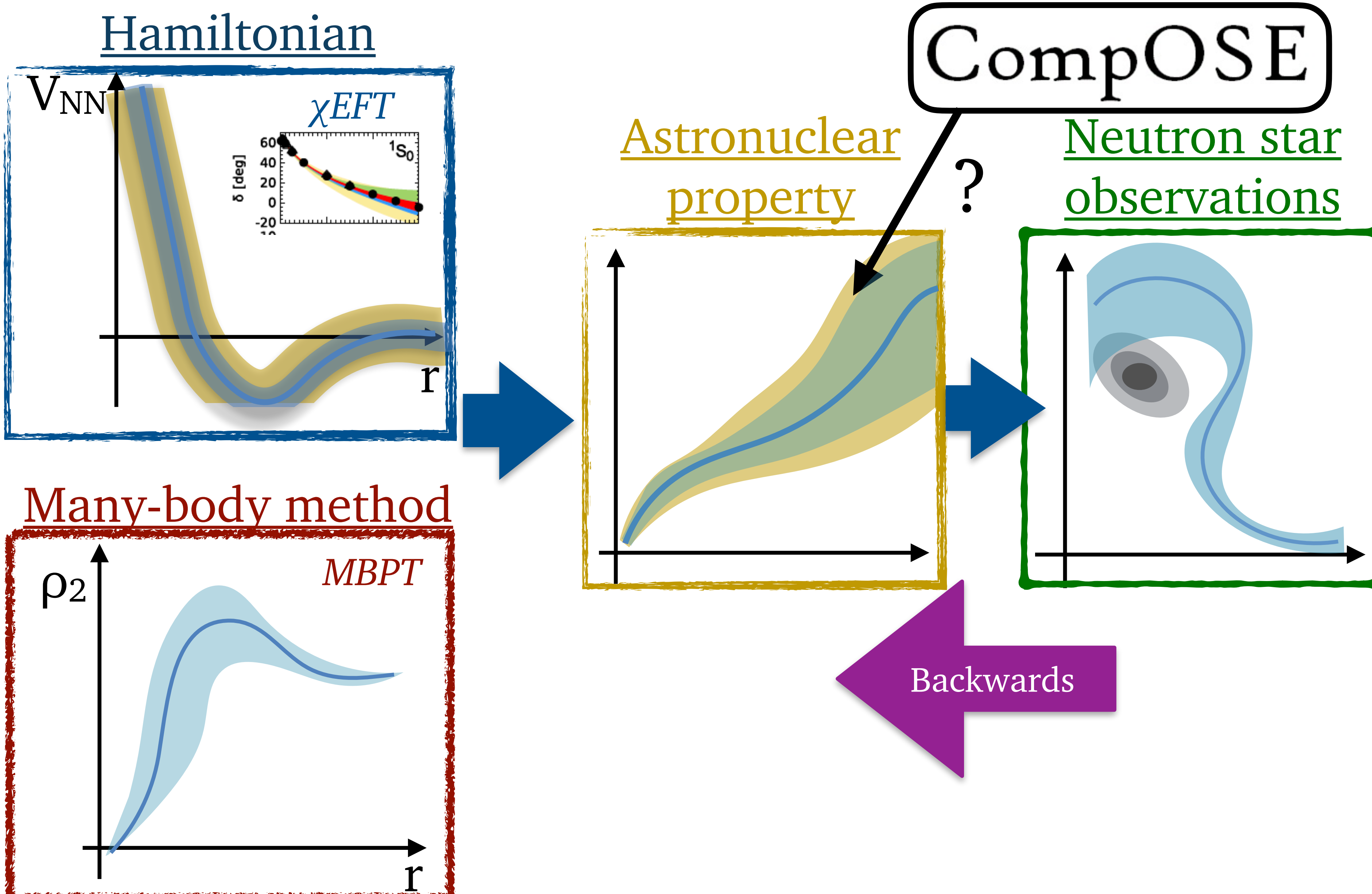
Nuclear error quantification



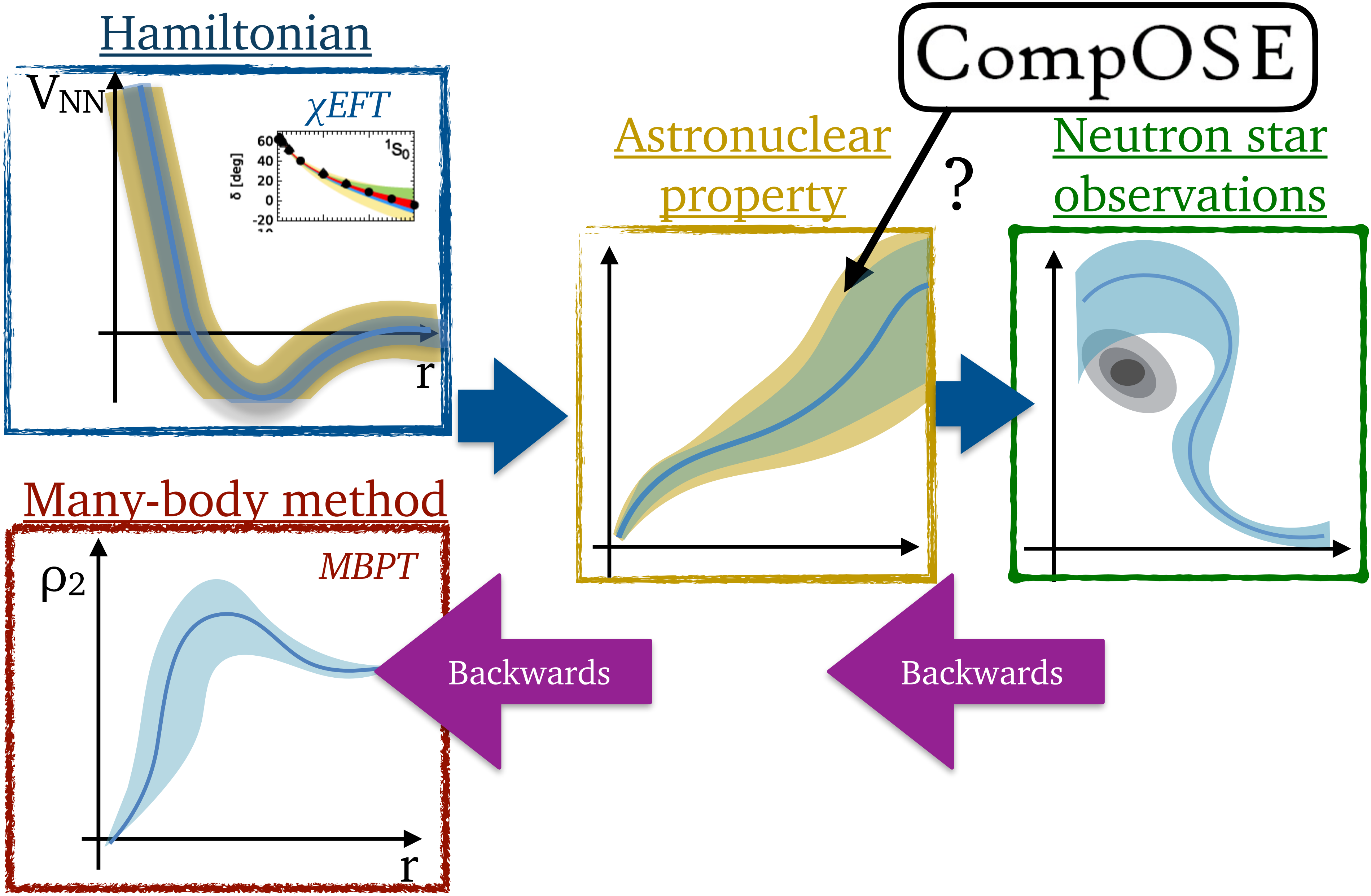
Nuclear error quantification



Nuclear error quantification



Nuclear error quantification



From M-R to EoS

Backwards

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi p r^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = \frac{4\pi}{c^2}\epsilon r^2$$

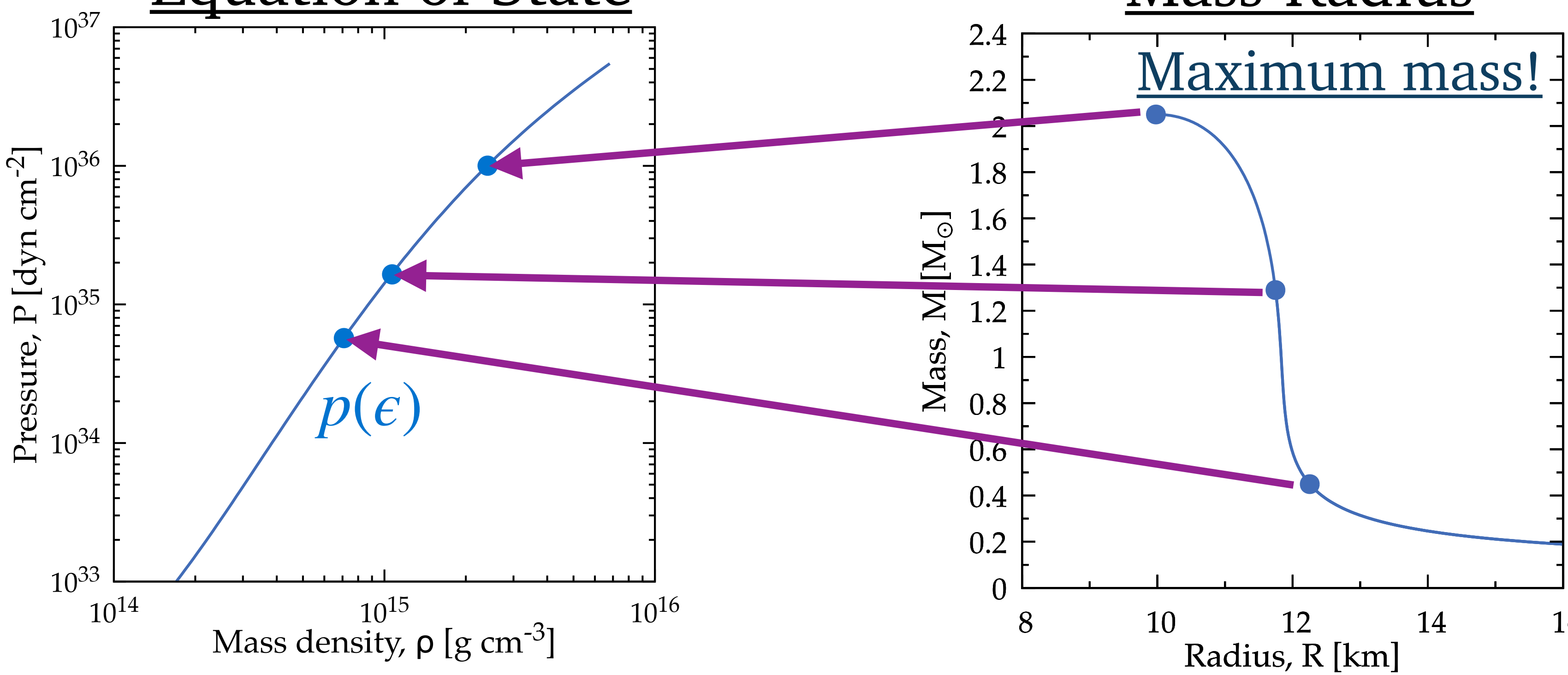
+

EoS

$$p \equiv p(\epsilon)$$

Equation of State

Mass-Radius



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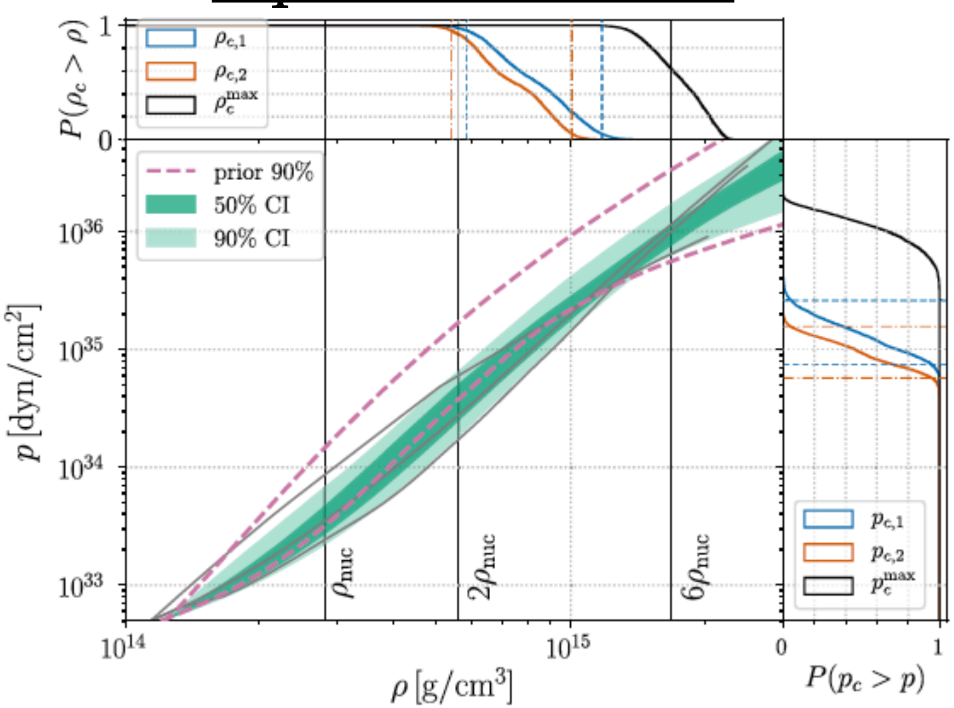
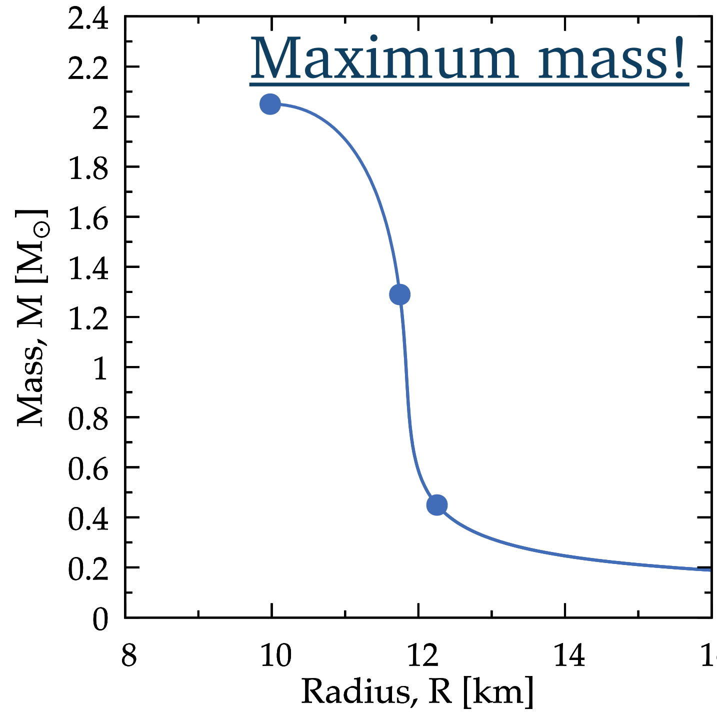


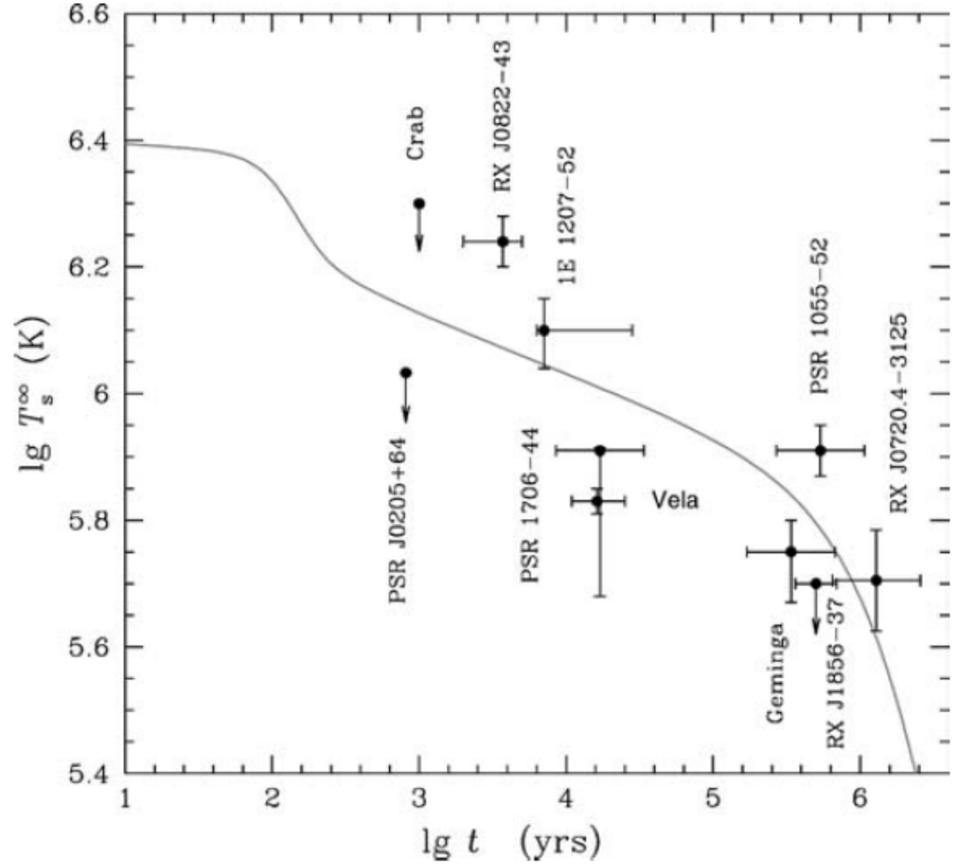
Mass-Radius



LIGO/VIRGO PRL 121 161101 (2018)

NS cooling

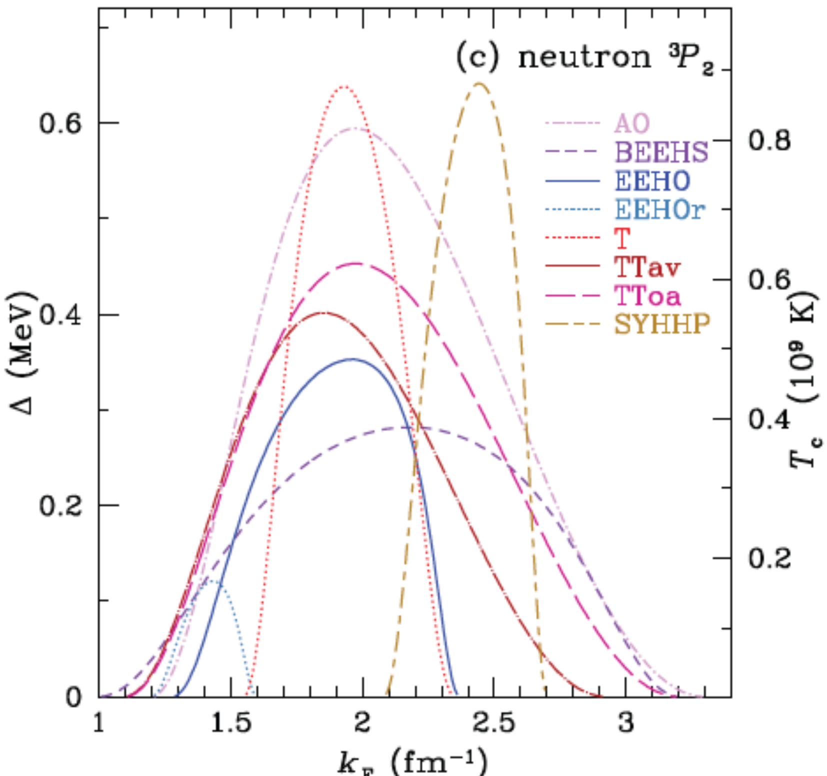
NS cooling curves ($M=1.3M_\odot$)



Yakovlev & Pethick, Annu. Rev. Astron. Astrophys. **42** 169 (2004)

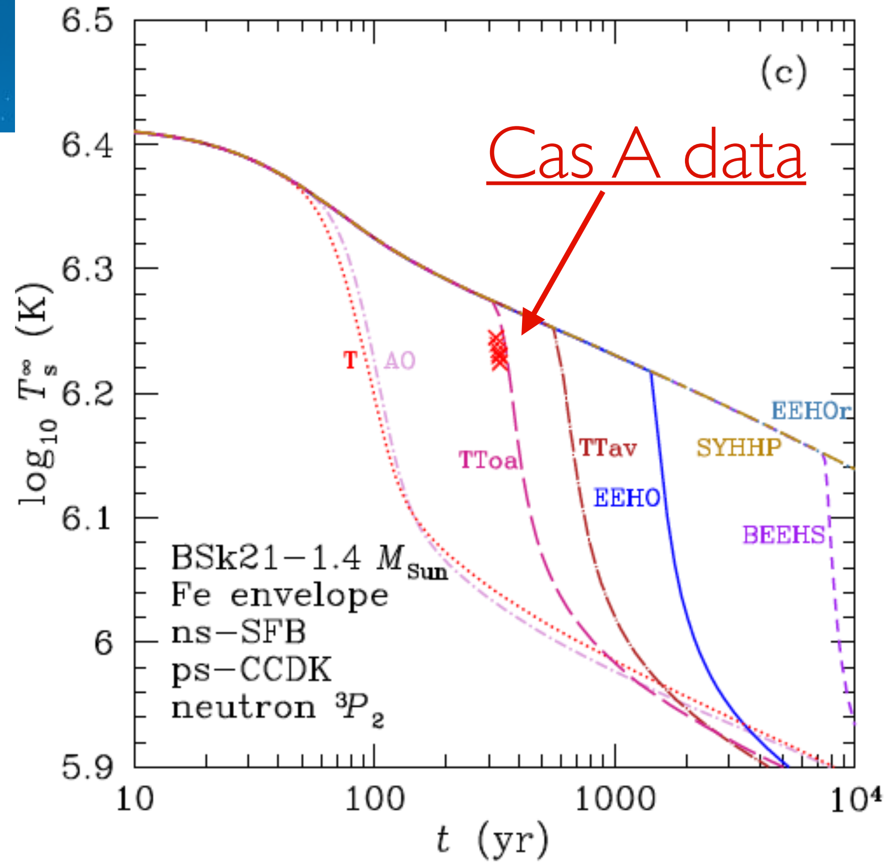
- Observational data available for a handful of NS
- Sensitive to **interior** physics
- Similar modelling for exotic scenarios (eg axion cooling)

Cooling of CasA



Ho, *et al.*, PRC **91** 015806 (2015)

Page, *et al.*, *PRL* **106** 081101 (2011)



Ingredients

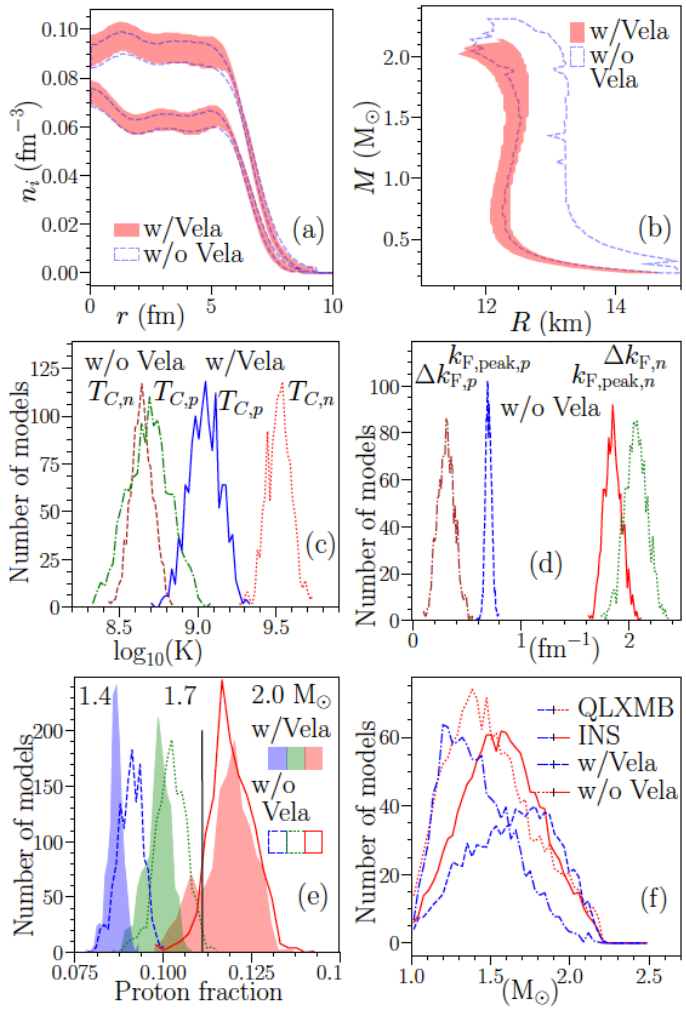
(a) Mass of pulsar
(b) EoS (determines radius)
(c) Internal composition
(d) **Pairing gaps** ($^1S_0$ & $^3PF_2$ channels)
(e) Atmosphere composition

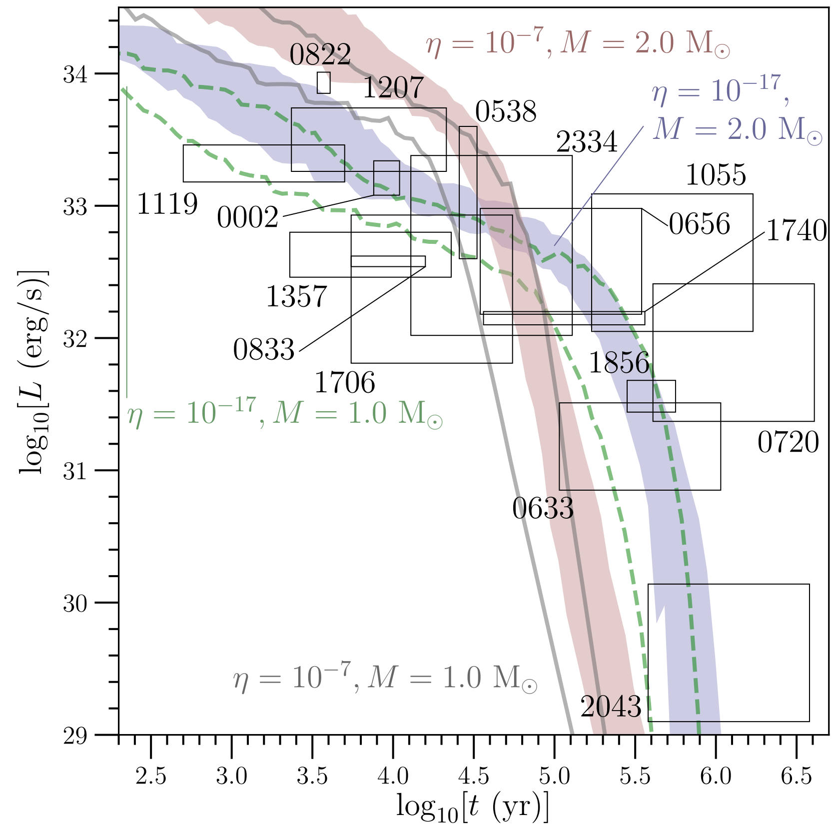
Name	Process	Emissivity (erg cm^{-3} s^{-1})
Modified Urca (neutron branch)	$n+n \to n+p+e^- + \bar{\nu}_e$ $n+p+e^- \to n+n+\nu_e$	$\sim 2\times10^{21} R T_9^8$
Modified Urca (proton branch)	$p+n \to p+p+e^- + \bar{\nu}_e$ $p+p+e^- \to p+n+\nu_e$	$\sim 10^{21} R T_9^8$
Bremsstrahlungs	$n+n \to n+n+\nu+\bar{\nu}$ $n+p \to n+p+\nu+\bar{\nu}$ $p+p \to p+p+\nu+\bar{\nu}$	$\sim 10^{19} R T_9^8$
Cooper pair	$n+n \to [nn]+\nu+\bar{\nu}$ $p+p \to [pp]+\nu+\bar{\nu}$	$\sim 5\times10^{21} R T_9^7$ $\sim 5\times10^{19} R T_9^7$
Direct Urca (nucleons)	$n \to p+e^- + \bar{\nu}_e$ $p+e^- \to n+\nu_e$	$\sim 10^{27} R T_9^6$

Backwards model for cooling

Backwards

$$T_c(k_F) = T_{c,\tau} e^{-\frac{1}{2}\left(\frac{k - k_{F,peak,\tau}}{\Delta k_{F,\tau}}\right)^2}$$





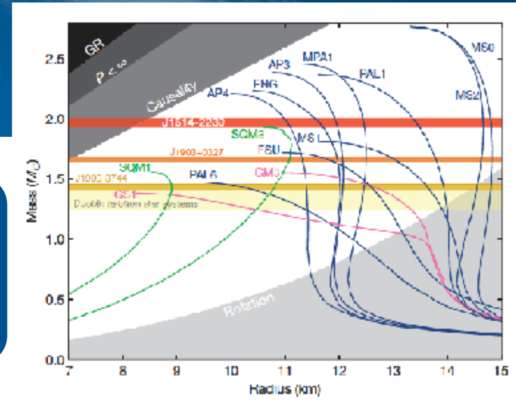
The 95% credible intervals for the luminosity-age curves for different neutron star masses and different amounts of light elements in the neutron star envelope (parameterized by eta), along with the luminosity and age values obtained from observations which were used in this work. Vela (labeled 0833) is not included.

Beloin, Han, Steiner *et al Phys. Rev. C 100, 055801 (2019)* [arXiv:1812.00494]
Beloin, Han, Steiner *et al Phys. Rev. C 97, 015804 (2018)* [arXiv:1612.04289]

Neutron star modelling

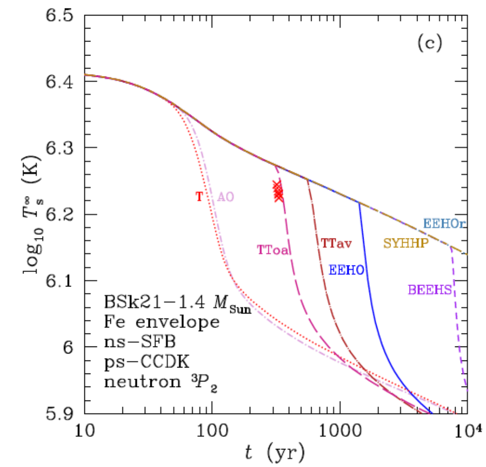
Input #1
EoS

→ **Observable #1**
Mass-Radius relation



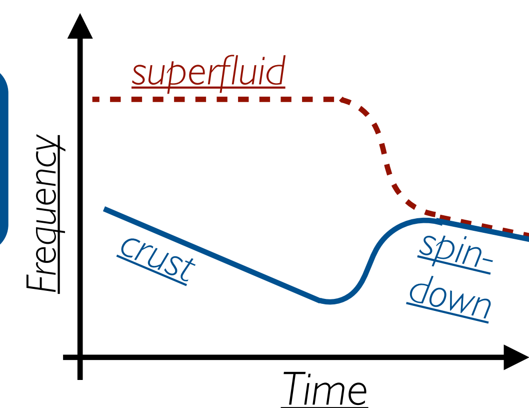
Input #2
Pairing gap

→ **Observable #2**
Cooling curve



Input #3
Crust-core

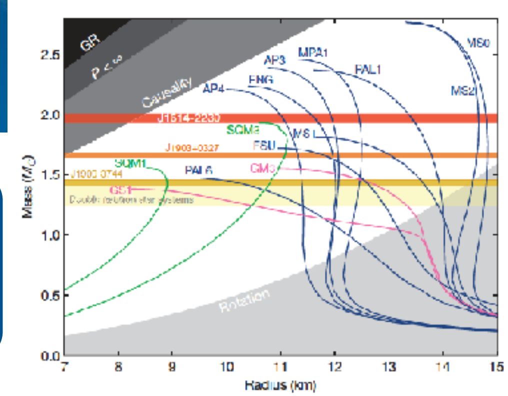
→ **Observable #3**
Glitching



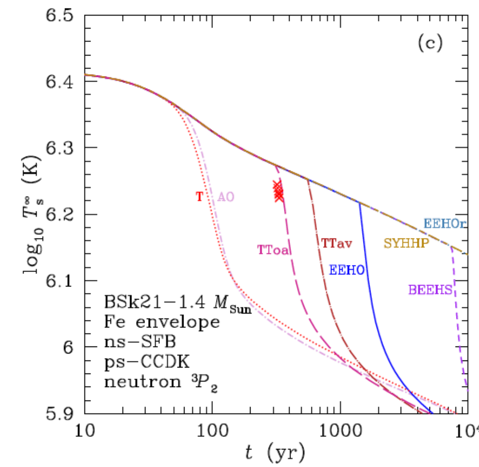
Neutron star modelling

Input
Consistent many-body theory

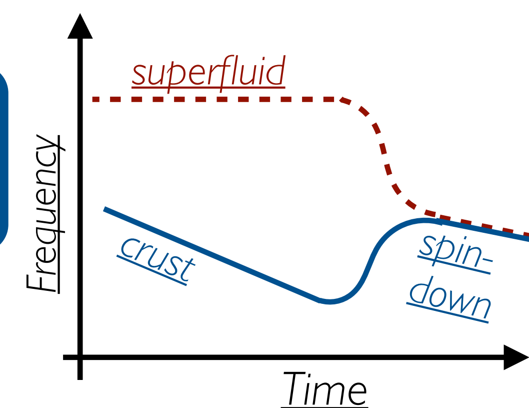
Observable #1
Mass-Radius relation



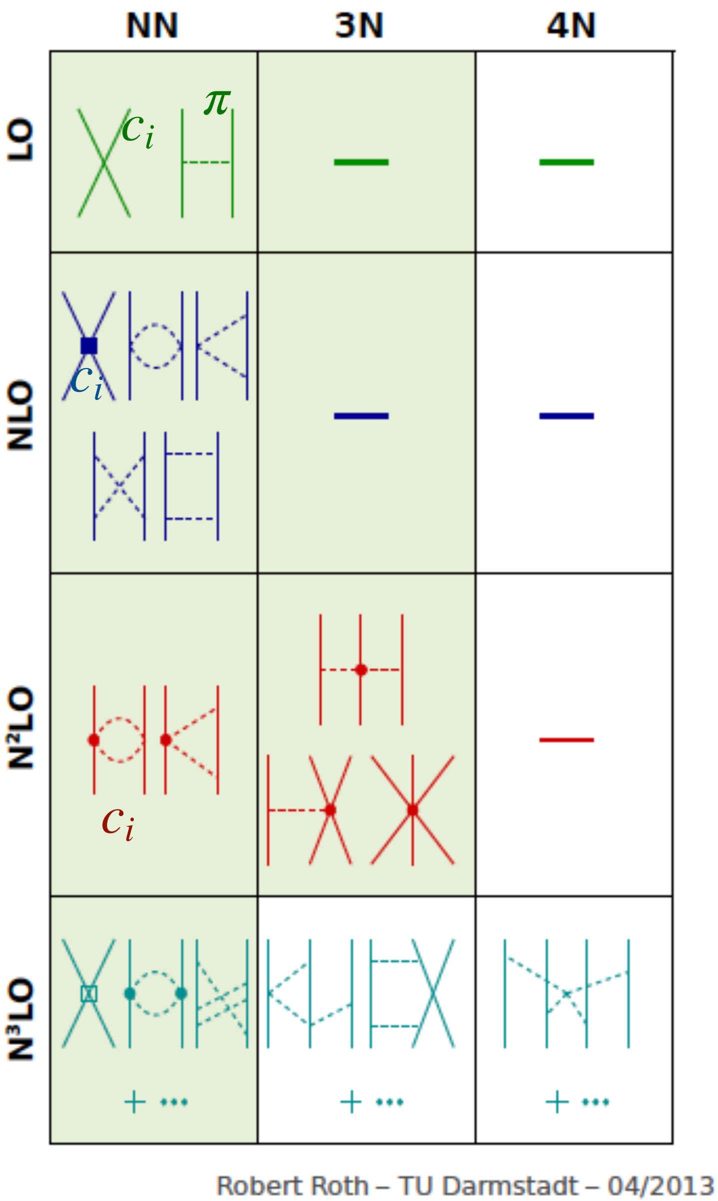
Observable #2
Cooling curve



Observable #3
Glitching



The compulsory χEFT slide

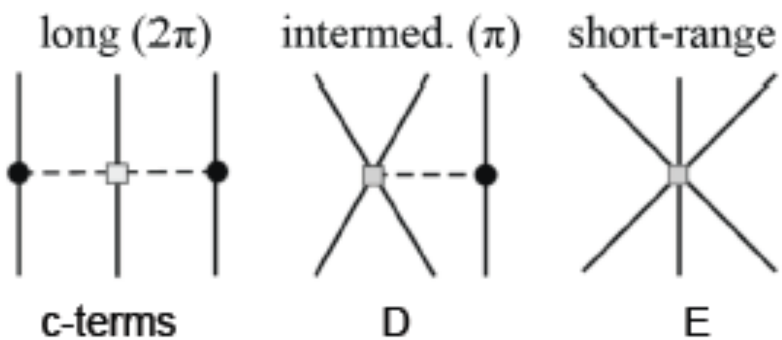


Chiral χEFT

- π and N as dof
- Systematic expansion
- 2N at N³LO - LECs from πN, NN
- 3N at N²LO - 2 more LECs
- (Often further renormalized)
- Error quantification **possible**

$$\mathcal{O}\left(\frac{Q}{\Lambda}\right)$$

$$\Lambda \sim 1 \text{ GeV}$$



Weinberg, *Phys. Lett. B* **251** 288 (1990), *NPB* **363** 3 (1991)
Entem & Machleidt, *PRC* **68**, 041001(R) (2003)
Tews, Schwenk *et al.*, *PRL* **110**, 032504 (2013)
Epelbaum, Frebs & Meissner, *PRL* **115**, 122301 (2015)

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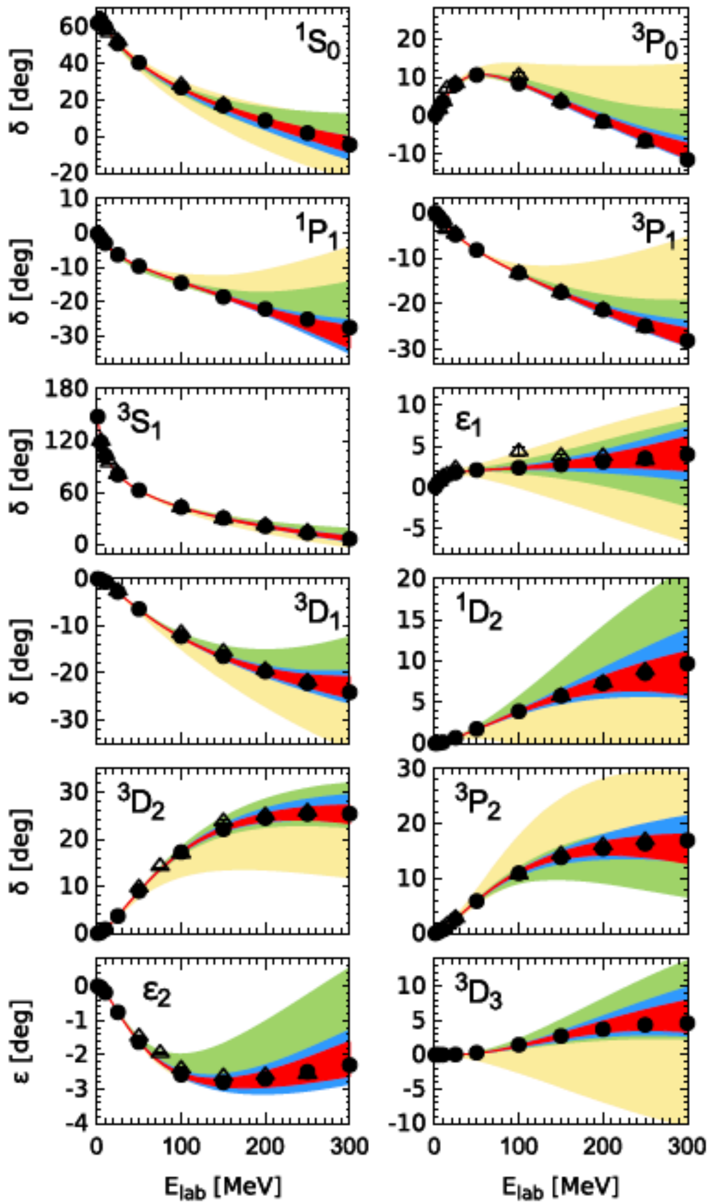

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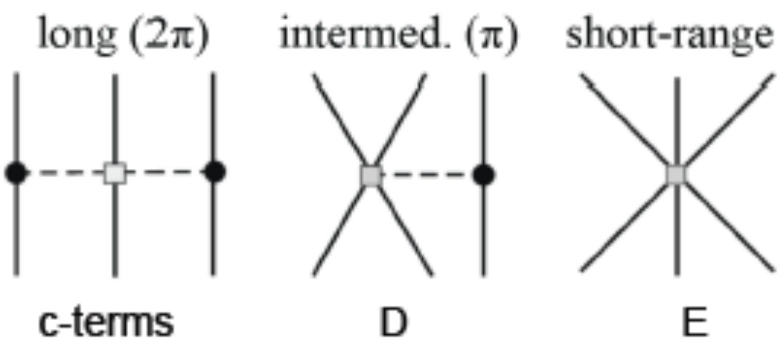

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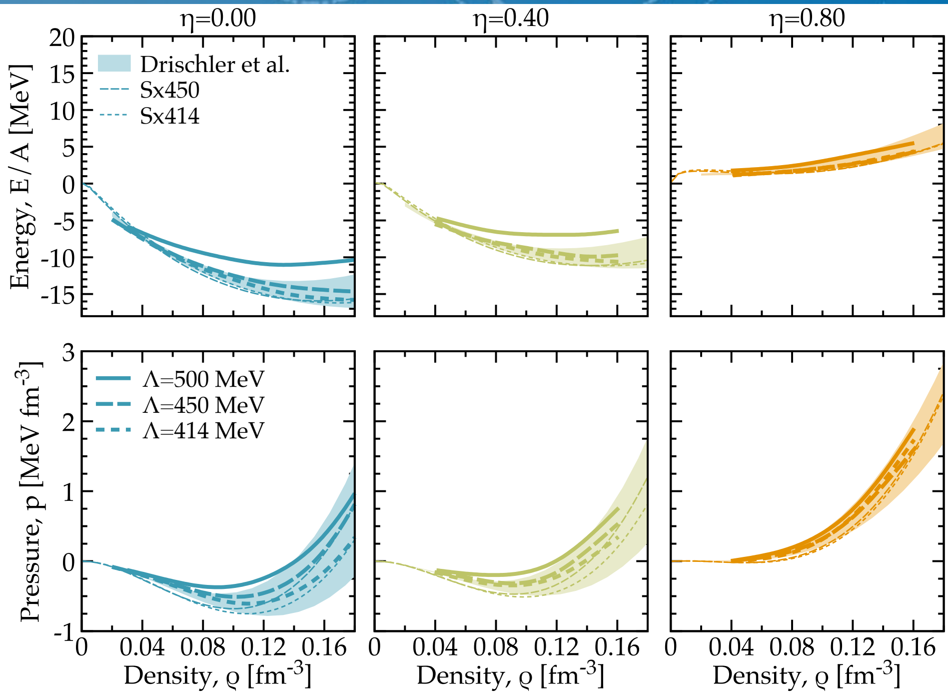
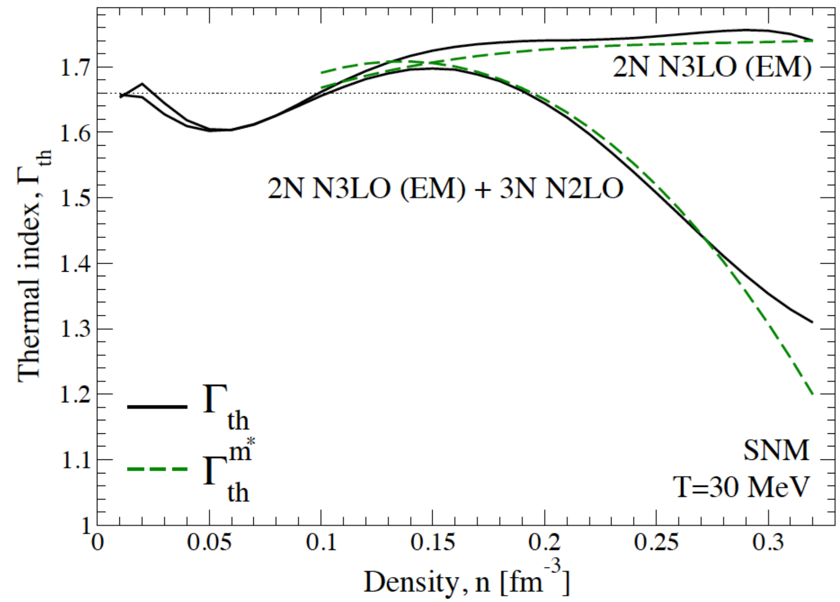
Self-Consistent Green's Functions



Rios, Frontiers Physics fphy.2020.00387 (2020) [arXiv:2006.10610]



Carbone & Schwenk, Phy. Rev C 100, 025805 (2019) [arXiv:1904.00924]

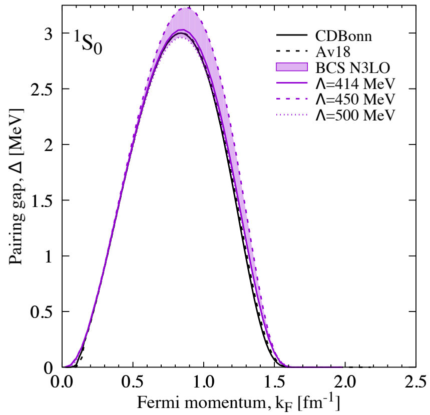
SCGF can treat:

- **Explicitly asymmetric matter ✔**
- Finite temperature ✔
- Systematic expansion ✔
- 3 nucleon forces ✔

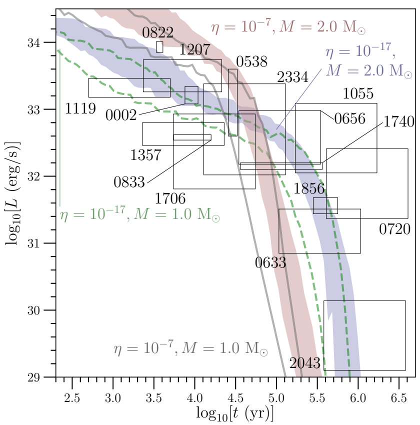
but:

- Numerically intensive
- Pairing?

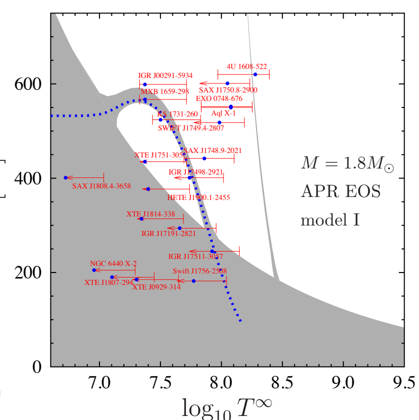
Why are pairing gaps necessary?



- Characterisation of **superfluidity**
  - Neutron superfluid, proton superconductor
  - Phase transitions



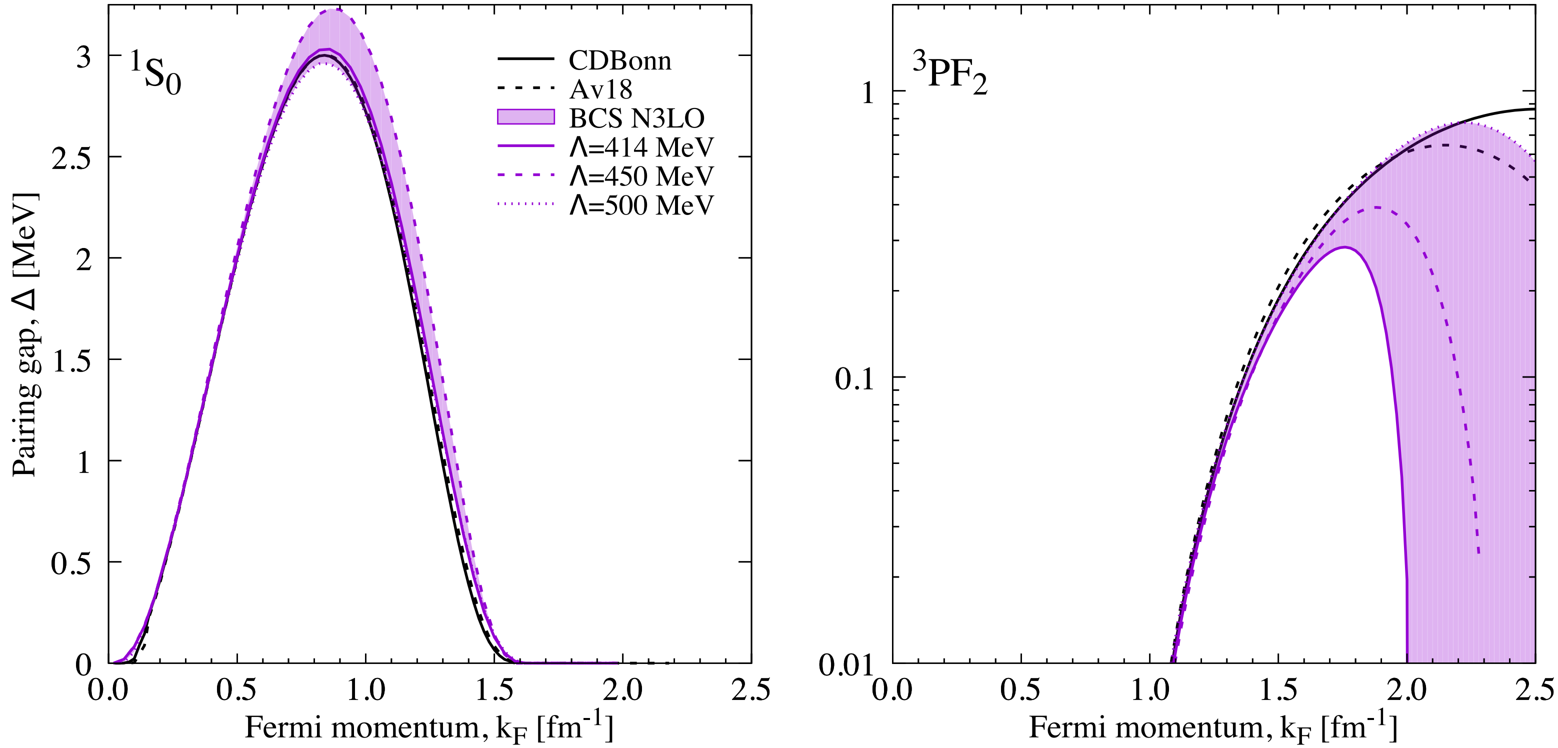
- Neutron star **cooling**
  - Neutrino rates through pair-breaking



- **r-mode** coupling
  - Superfluidity & viscosity

Kantor et al. Phys. Rev. Lett. **125** 151101 (2020)

Bardeen-Cooper-Schrieffer pairing

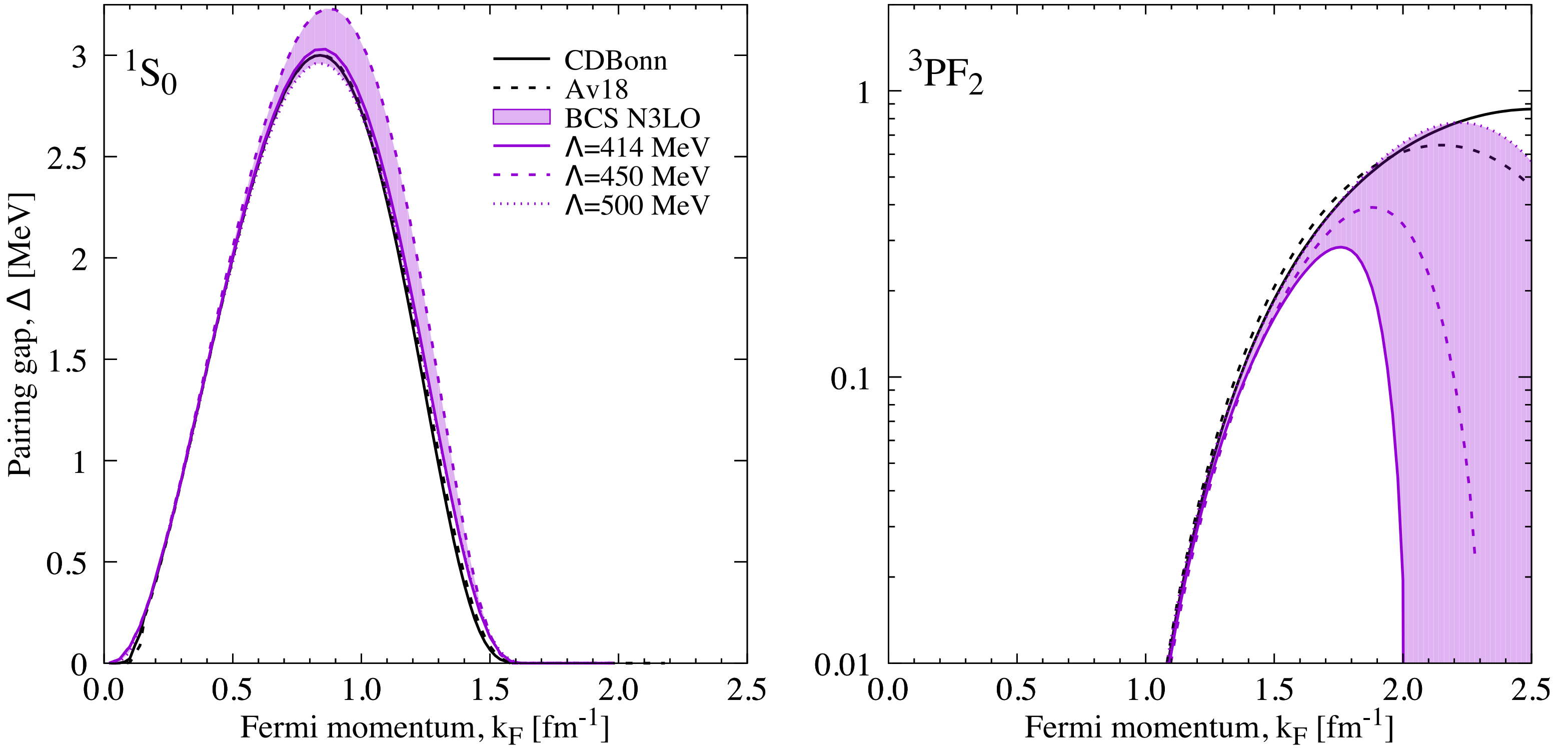


BCS equation

$$\boxed{\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k|V_{nn}^{LL'}|k'\rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad + \quad \chi_k = \varepsilon_k - \mu}$$

- Single-particle spectrum choice: $\varepsilon_k = \frac{k^2}{2m} + U(k) - \mu$
- Angular gap dependence: $|\Delta_k|^2 = \sum_L |\Delta_k^L|^2$

Bardeen-Cooper-Schrieffer pairing



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- Single-particle spectrum choice: $\varepsilon_k = \frac{k^2}{2m} + \cancel{U}(k) - \mu$
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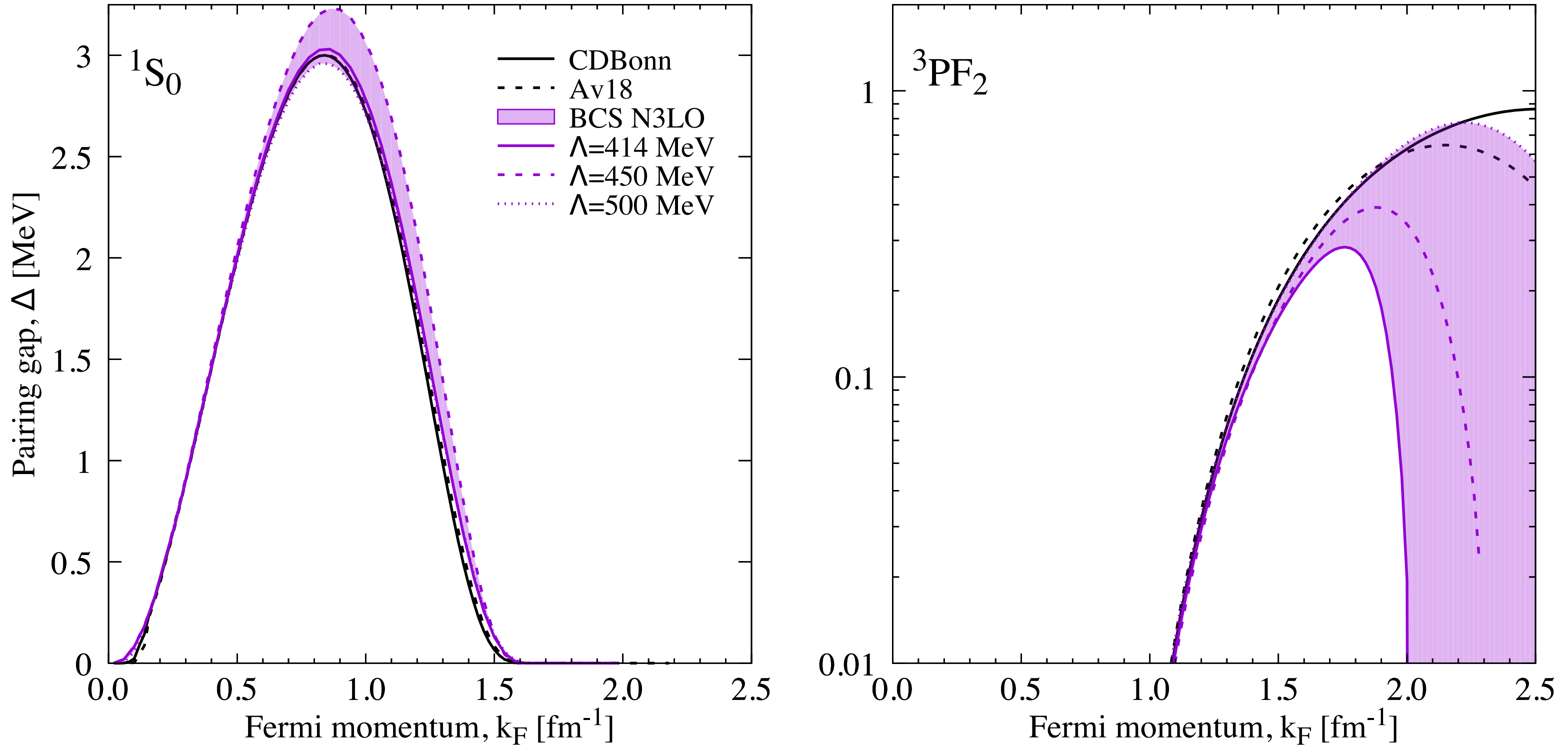

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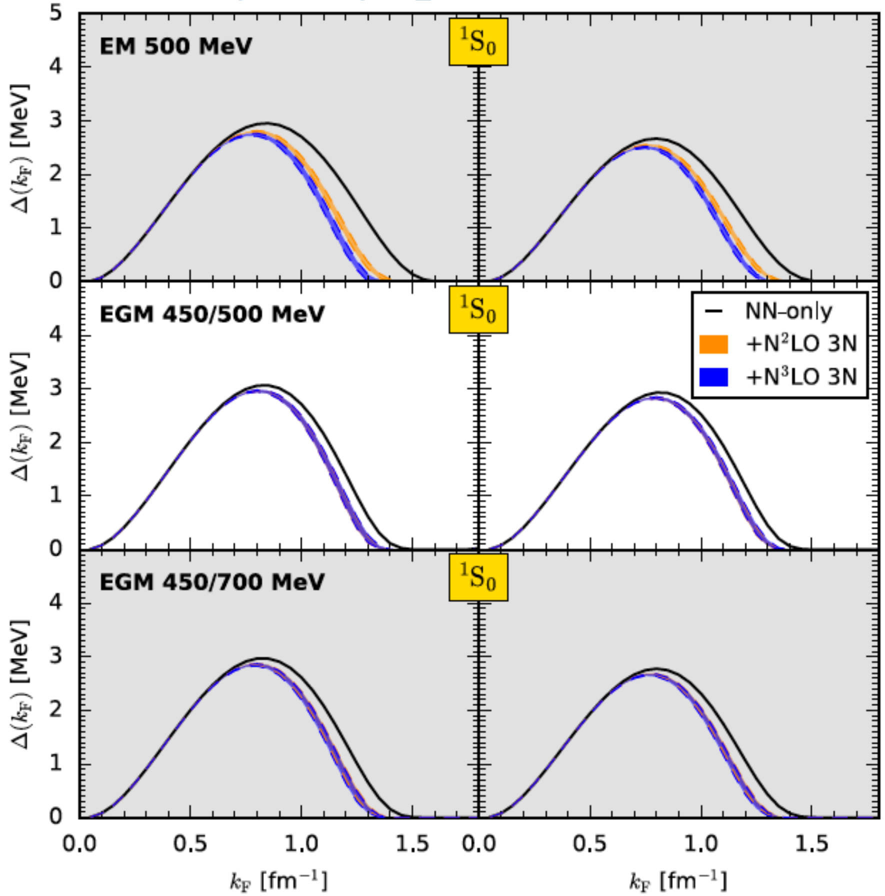
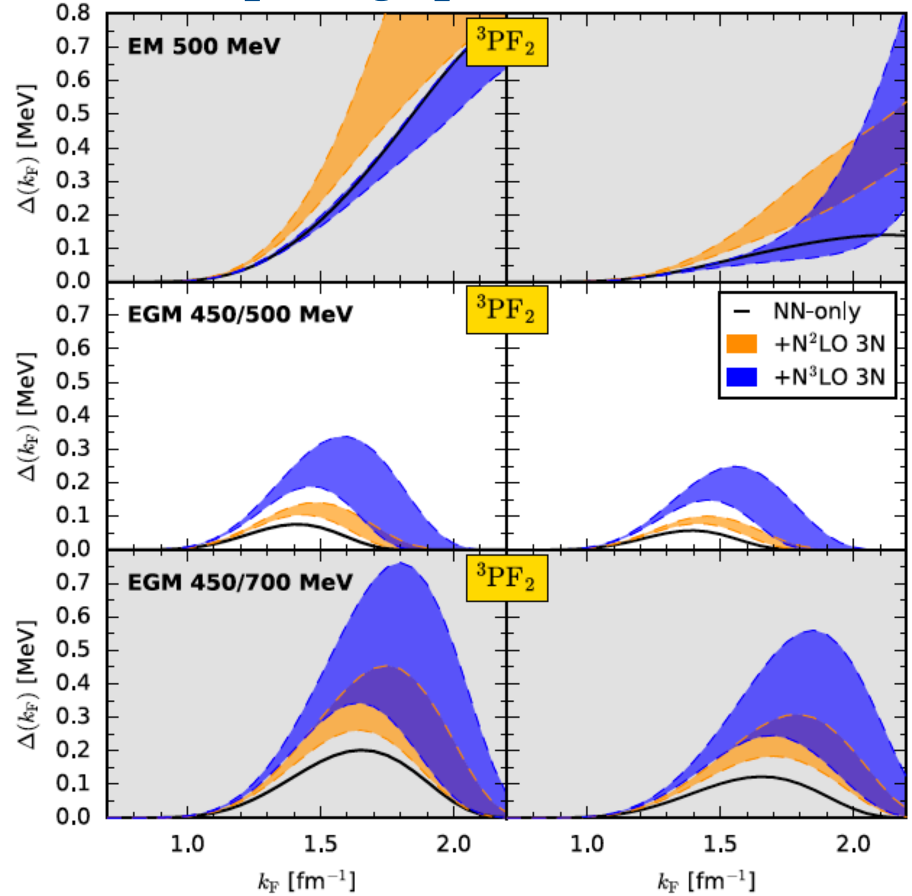
- Single-particle spectrum choice: $\varepsilon_k = \frac{k^2}{2m} + \cancel{U}(k) - \mu$
- Angular gap dependence: $|\Delta_k|^2 = \sum_L |\Delta_k^L|^2 \approx |\Delta_k^L|^2$

BCS+HF gaps in neutron matter

Singlet gaps with 3NF



Triplet gaps with 3NF

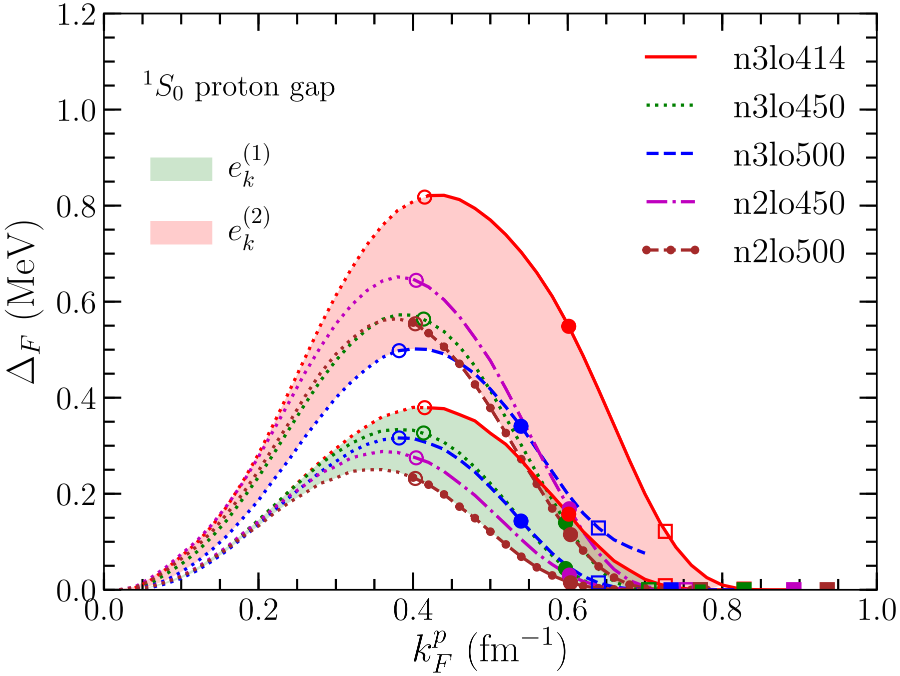


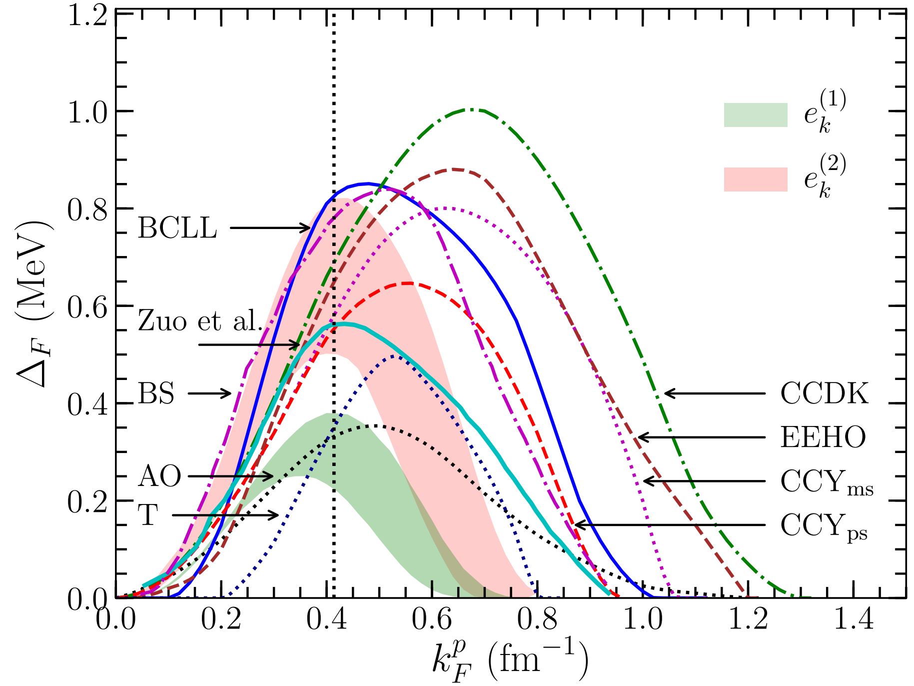
- **Error estimates** from chiral expansion are **sophisticated**
- Many-body treatment uncertainty?

Drischler, Kruger, Hebeler, Schwenk, *Phys Rev C* **95** 024302 (2017) [arXiv:1610.05213]

BCS estimates of proton gaps

Proton pairing gap

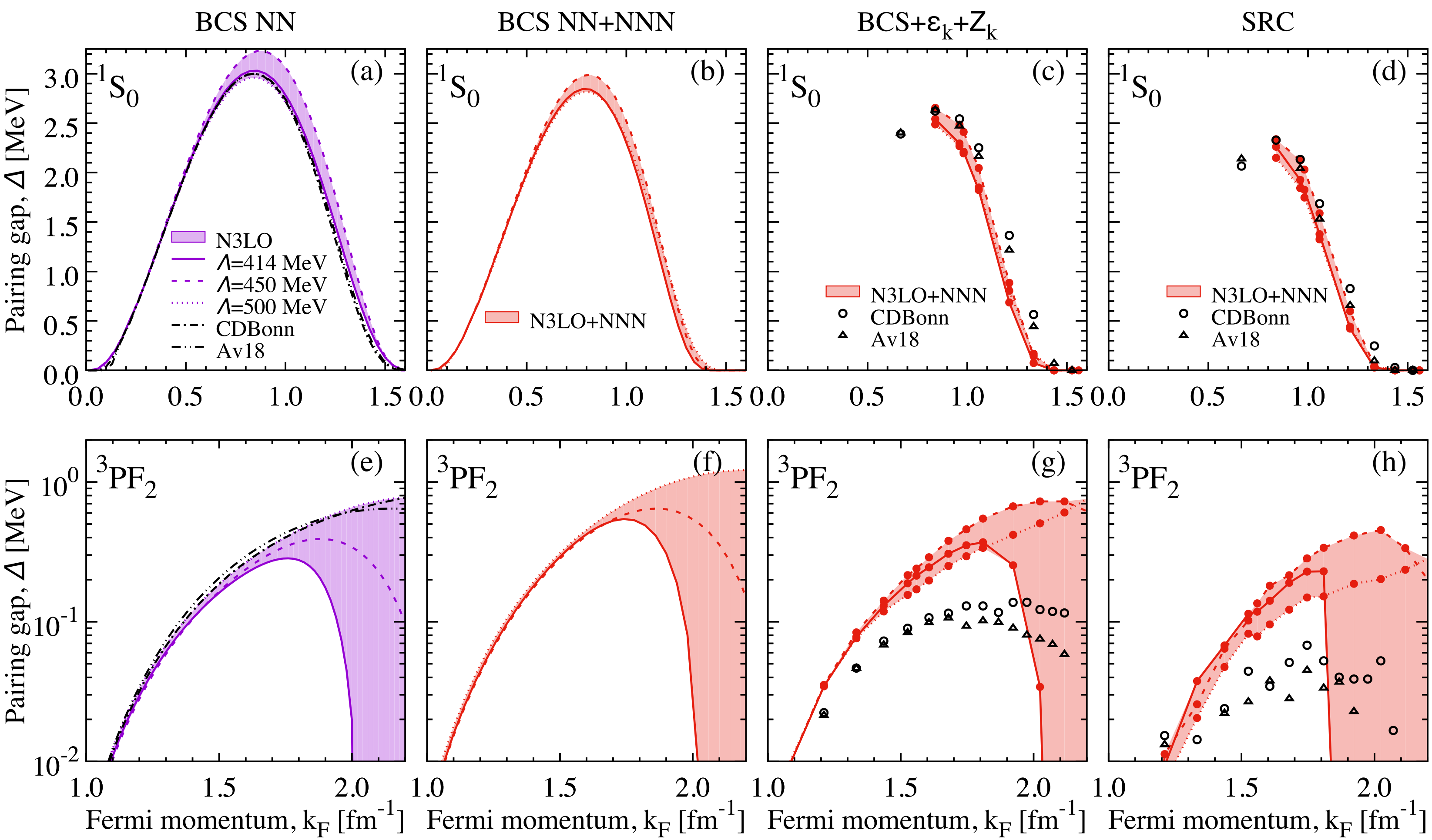




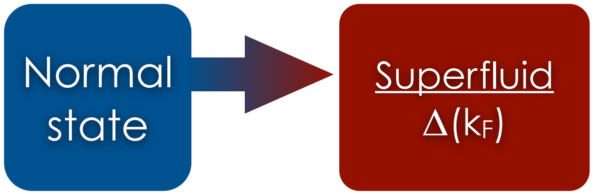
Lim & Holt [arxiv:1709.08793]

- Single-particle spectrum choice: $\varepsilon_k = \dfrac{k^2}{2m} + U(k) - \mu$
  - **Consistently** computed at different orders in χEFT
  - Uncertainty from **EoS** & **spectrum** at 1st and 2nd order

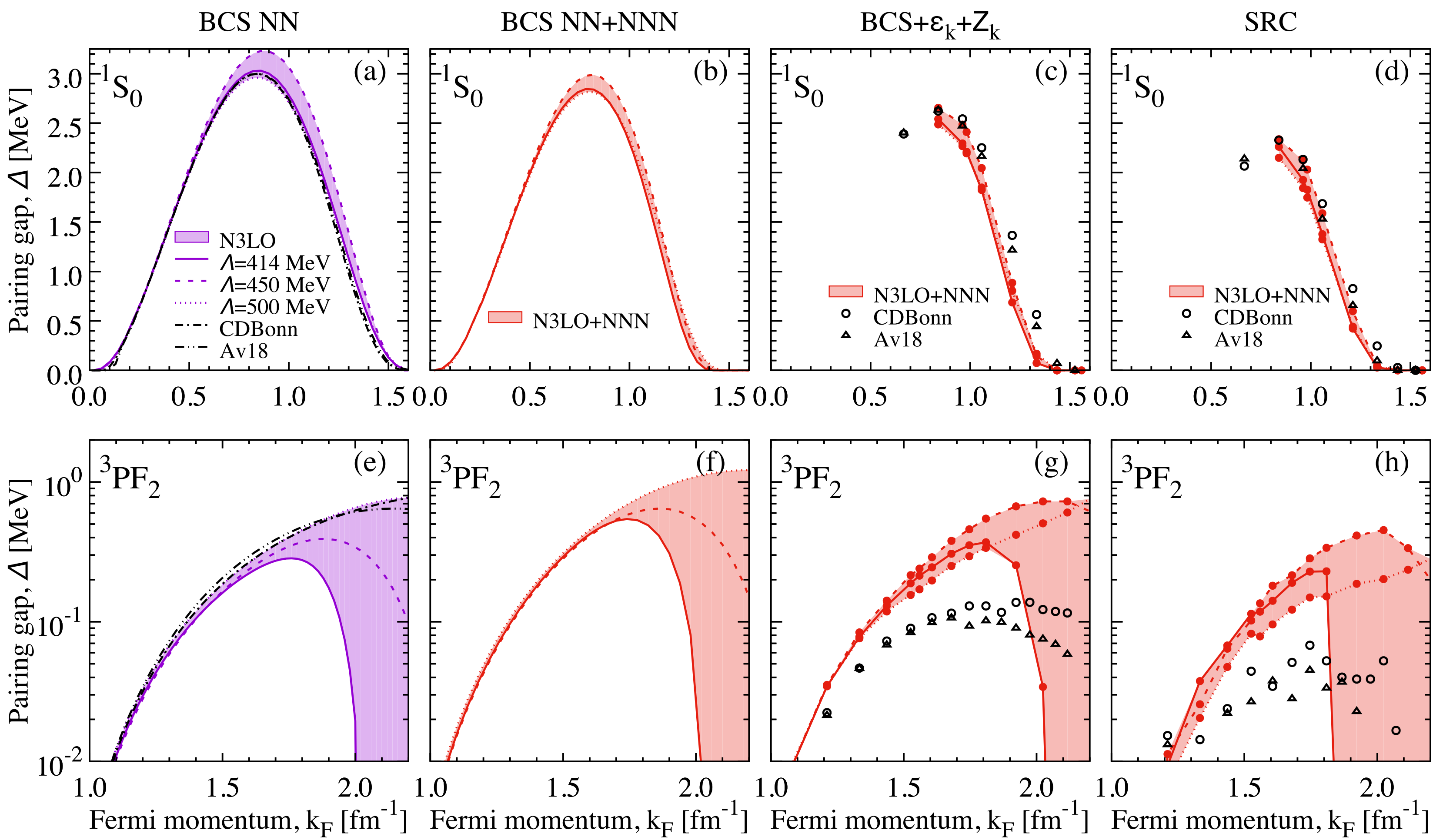
Beyond-BCS: short-range correlations



A. Rios, Dickhoff, Polls, JLTP **189**, 234 (2017) [arXiv:1707.04140]

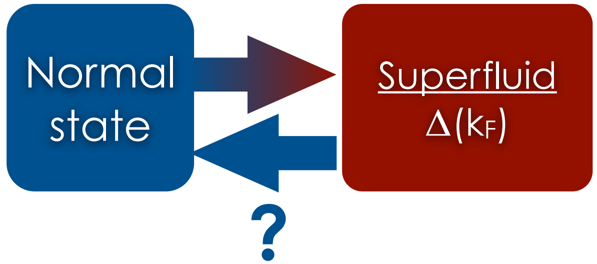


Beyond-BCS: short-range correlations

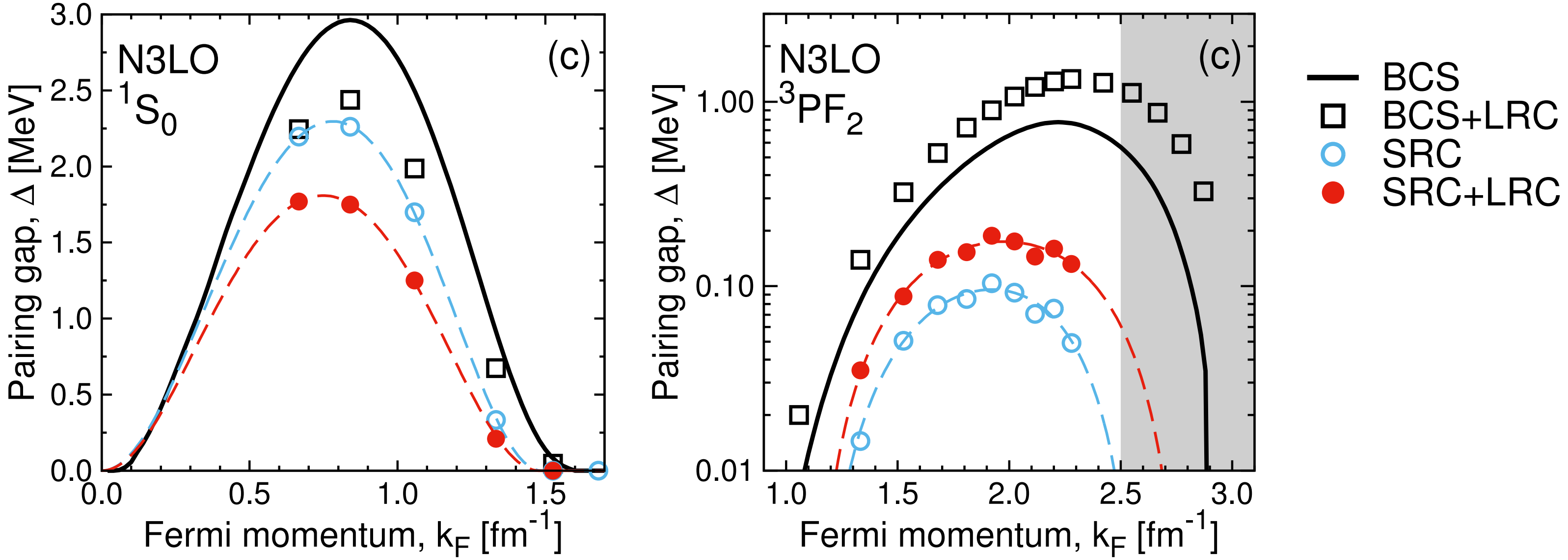


A.

Rios, Dickhoff, Polls, JLTP **189**, 234 (2017) [arXiv:1707.04140]



Gap parametrization example



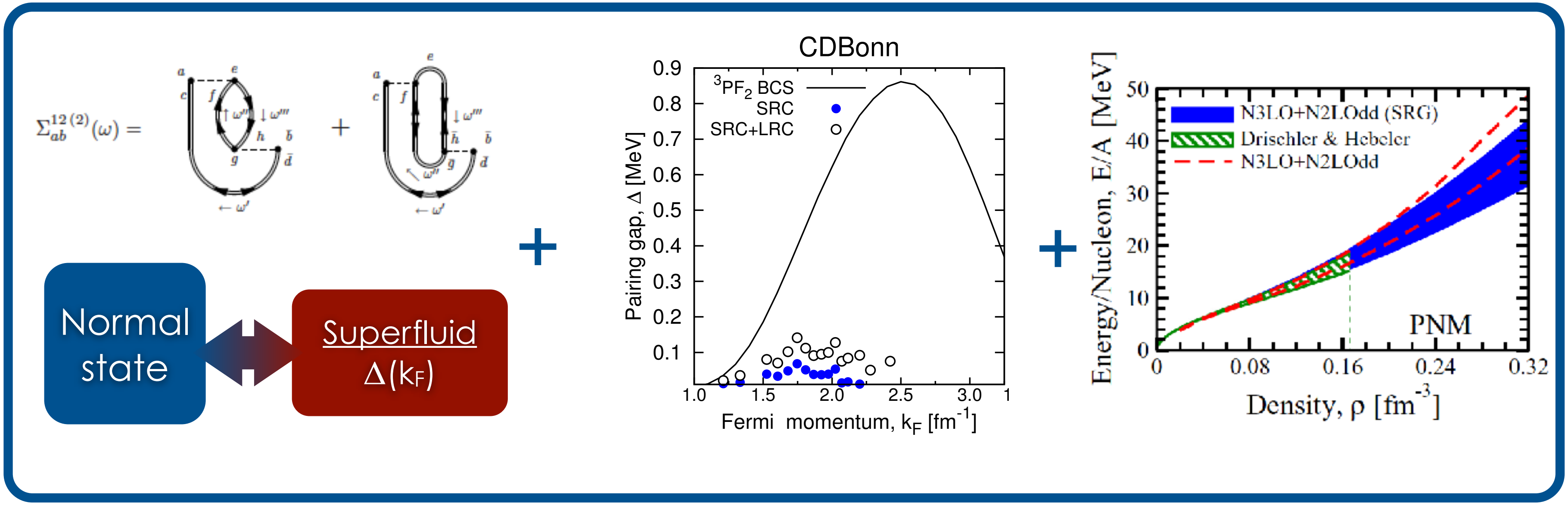
Ding, Rios, et al Phys. Rev. C 94, 025802 (2016) [arXiv:1601.01600]

$$\boxed{\Delta_L^{JST}(k_F) = \Delta_0 \frac{(k_F - k_0)^2}{(k_F - k_0)^2 + k_1} \frac{(k_F - k_2)^2}{(k_F - k_2)^2 + k_3}}$$

Kaminker, Haensel, Yakovlev, AA **373** L17 (2001) [arXiv:astro-ph/0105047]
Ho, Elshamouty, Heinke, Potekhin, Phys. Rev. C 91, 015806 (2015) [arXiv:1412.7759]

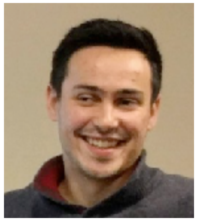
		Δ_0	k_0	k_1	k_2	k_3
Singlet	Av18	14.07	0.04	1.00	1.44	0.78
	N3LO	5.85	0	0.46	1.48	0.42
Triplet	Av18	0.17	1.1	0.35	2.18	0.05
	N3LO	0.6	1.11	0.69	2.79	0.53

How to go beyond BCS?



Nambu-covariant SCGF technique

- Pairing correlations ✔
- Finite temperature ✔
- Systematic expansion ✔
- 3 nucleon forces ✔



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Nambu fields

- $\mathscr{B}$ and $\bar{\mathscr{B}} \equiv$ orthonormal bases
- Let $\bar{\cdot}$ be the involution $|b\rangle \to |\bar{b}\rangle$
- Define $\mu \equiv (b, g)$ and $\bar{\mu} \equiv (\bar{b}, \bar{g})$ where $g \in \{1,2\}$
- Then Nambu fields are defined as

$$\mathrm{A}^{\mu} \equiv \mathrm{A}^{(b,g)} \equiv \begin{pmatrix} a_b \\ a^{\dagger}_{\bar{b}} \end{pmatrix}_g$$

$$\mathrm{A}^{\dagger}_{\mu} \equiv \mathrm{A}^{\dagger}_{(b,g)} \equiv \begin{pmatrix} a^{\dagger}_b & a_{\bar{b}} \end{pmatrix}_g$$

- Canonical anticommutation relation
$$\{\mathrm{A}^{\mu}, \mathrm{A}^{\nu}\} = \delta_{\mu\bar{\nu}} \;\; , \;\; \{\mathrm{A}^{\dagger}_{\mu}, \mathrm{A}^{\dagger}_{\nu}\} = \delta_{\mu\bar{\nu}} \;\; , \;\; \{\mathrm{A}^{\mu}, \mathrm{A}^{\dagger}_{\nu}\} = \delta_{\mu\nu}$$

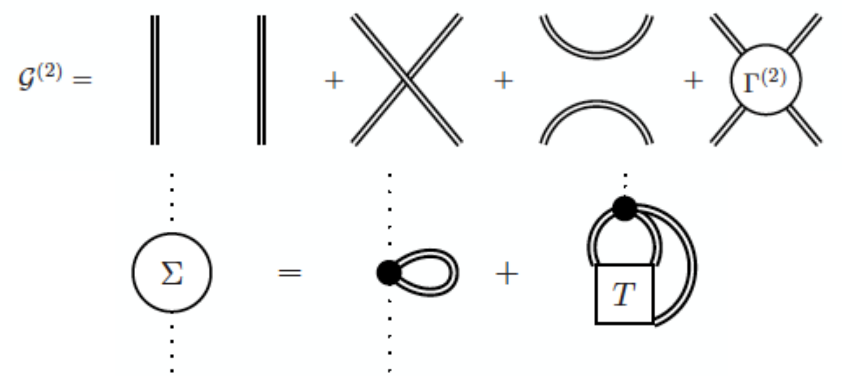
Mixed one-body Green's function

- Mixed propagator
$$-\mathscr{G}^{\mu}{}_{\nu}(\tau, \tau') \equiv \left\langle \mathrm{T}\left[\mathrm{A}^{\mu}(\tau)\mathrm{A}^{\dagger}_{\nu}(\tau')\right]\right\rangle \;\; , \;\; \text{with } \langle\, .\,\rangle = \mathrm{Tr}\left(\,.\,\rho\right) \text{ and } \rho \equiv \frac{e^{-\beta\Omega}}{\mathrm{Tr}\left(e^{-\beta\Omega}\right)}$$
  - Heisenberg representation + imaginary-time evolution
  - Grand canonical ensemble
  - (1,1)-tensor
- In the single-particle basis one recovers the usual one
$$-\mathscr{G}^{(b,g)}{}_{(c,h)}(\tau, \tau') = \begin{pmatrix} \left\langle \mathrm{T}\left[a_b(\tau)a^{\dagger}_c(\tau')\right]\right\rangle & \left\langle \mathrm{T}\left[a_b(\tau)a_{\bar{c}}(\tau')\right]\right\rangle \\ \left\langle \mathrm{T}\left[a^{\dagger}_{\bar{b}}(\tau)a^{\dagger}_c(\tau')\right]\right\rangle & \left\langle \mathrm{T}\left[a^{\dagger}_{\bar{b}}(\tau)a_{\bar{c}}(\tau')\right]\right\rangle \end{pmatrix}_{gh}$$

Dyson equation

- Partitioning considered
$$\Omega = \underbrace{\frac{1}{2!}\sum_{\mu\nu} U_{\mu\nu}\, \mathrm{A}^{\mu}\mathrm{A}^{\nu}}_{\Omega_0} + \underbrace{\frac{1}{4!}\sum_{\alpha\beta\gamma\delta} v^{(2)}_{\alpha\beta\gamma\delta}\, \mathrm{A}^{\alpha}\mathrm{A}^{\beta}\mathrm{A}^{\gamma}\mathrm{A}^{\delta}}_{\Omega_1}$$
- Dyson equation [Dyson, 1949] [Schwinger, 1951]
$$\mathscr{G}^{\mu\nu}(\omega_n) = \mathscr{G}^{(0)\mu\nu}(\omega_n) + \sum_{\lambda_1\lambda_2} \mathscr{G}^{(0)\mu\lambda_1}(\omega_n)\, \Sigma_{\lambda_1\lambda_2}(\omega_n)\, \mathscr{G}^{\lambda_2\nu}(\omega_n)$$

Diagrammatic expansion



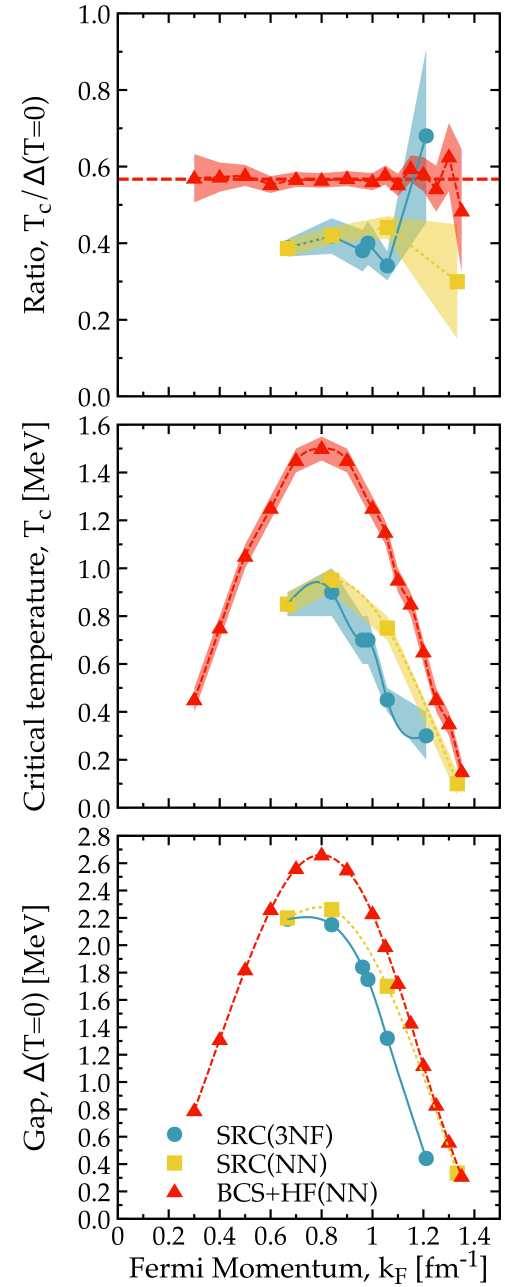
Why go beyond BCS?

BCS predictions:

Singlet $^1S_0$ $$\frac{T_c}{\Delta(T=0)} = 0.567$$

Triplet $^3PF_2$ $$\frac{T_c}{\Delta(T=0)} = \begin{cases} 0.842, m = 0 \\ 0.493, \quad m = \pm 2 \end{cases}$$

D.G. Yakovlev et al., Phys. Rep. **354** 155 (2001)

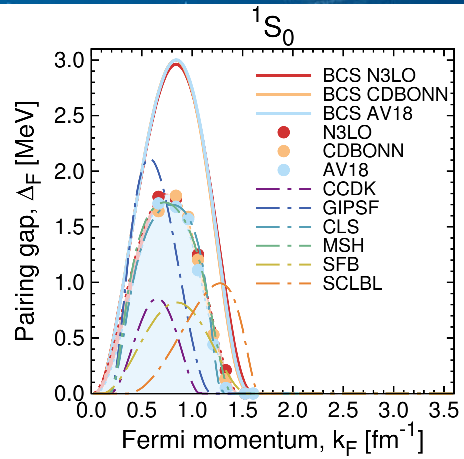


Beyond BCS:

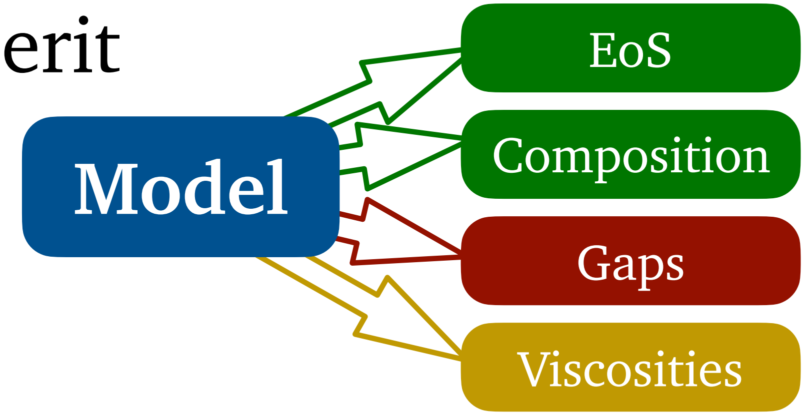
1) Numerically perform simulations
2) Compute different T's & locate closure
3) Take ratio

Wish list for COMPOSE

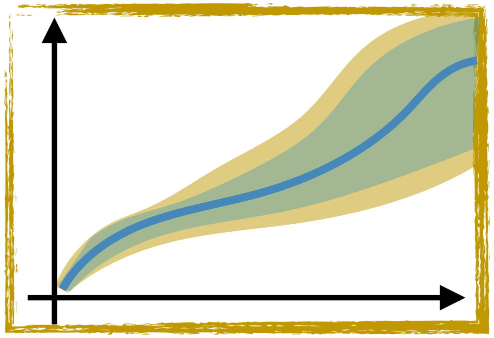
1) Pairing gaps ($^1S_0$ & $^3PF_2$ ideally)

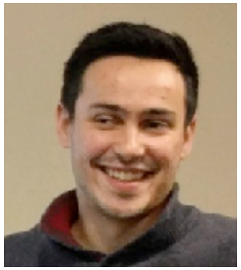


2) "Consistency" figure of merit



3) Uncertainties in predictions?





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Thank you!

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