Density Functional Theory for Compact Stars

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Goals:

- Construct an EoS in a form of density functional: the pressure of dense zero-temperature matter is a functional of energy-density: $P(\varepsilon(r))$
- The parameters of the functional are adjusted to the available data; in our case astrophysics and laboratory data.
- *Ab initio* calculations are data $\rightarrow$ check compatibility and adjust if required.
- DFT must be versatile enough to accommodate the baryon spin-1/2 octet and spin-3/2 decouplet.
- Fast in implementation to generate quickly families of EoS

DFT's :

- Relativistic mean-field models of nuclear matter reinterpreted as DFT:
  (a) relativistic covariance, causality is fulfilled automatically (+)
  (b) The Lorentz structure of interactions is maintained explicitly (+)
  (c) straightforward extension to the strange sector and resonances (+)
  (d) fast implementation (+)
  (e) the microscopic counterpart is unknown [not a QFT in the QED/QCD sense] (-)
- Non-relativistic DFTs (e.g. Skyrme or Gogny classes):
  (a) high accuracy at low-densities (+)
  (b) extensive tests on laboratory nuclei (+)
  (c) relativistic covariance is lost and high-density extrapolation is not obvious (-)
  (d) extensions to heavy baryons not straightforward (-)

This talk

- Relativistic covariant DFT based on the DDME-2 parametrization and its variants
- Constraints from laboratory, astrophysics and *ab initio* calculations
- Examples of implementation in astrophysics of compact stars:

  (a) Equation of state, $M - R$ relation and deformability (GW170817)

  (b) First order phase transition to quark phase(s)

  (c) Rapid rotation and large masses (GW190814)

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References:

- Equation of state: Eur. Phys. J. A 54, 133 (2018)
  Phys. Lett. B 783, 234, (2018)
  Phys. Rev. C 100, 015809 (2019)
- Deformabilities: Astrophys. J. Lett 874, L22 (2019)
- QCD-phase transition: Phys. Rev. D 101, 063022 (2020)
- Rapid rotation: Phys. Rev. D 102, 041301 (2020)
  Phys. Lett. B 810, 135812 (2020)

Nuclear matter Lagrangian:

$$\begin{aligned}
\mathcal{L}_{NM} &= \underbrace{\sum_B \bar{\psi}_B \left[\gamma^\mu \left(i\partial_\mu - g_{\omega BB}\omega_\mu - \frac{1}{2} g_{\rho BB} \boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\right) - (m_B - g_{\sigma BB}\sigma)\right]\psi_B}_{\text{baryons}} \\
&+ \underbrace{\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu}_{\text{mesons}} \\
&- \underbrace{\frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}^\mu\cdot\boldsymbol{\rho}_\mu}_{\text{mesons}} + \underbrace{\sum_\lambda \bar{\psi}_\lambda(i\gamma^\mu\partial_\mu - m_\lambda)\psi_\lambda}_{\text{leptons}} - \underbrace{\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{electromagnetism}} ,
\end{aligned}$$

- B-sum is over the baryonic octet $B \equiv p, n$
- Meson fields include σ meson, $\boldsymbol{\rho}_\mu$-meson and ω_μ-meson
- Leptons include electrons, muons and neutrinos for $T \neq 0$

Two types of relativistic density functionals based on relativistic Lagrangians

- linear mesonic fields, density-dependent couplings (DDME2, DD2, etc.)
- non-linear mesonic fields; coupling constant are just numbers (NL3, GM1-3, etc.)

Fixing the couplings: nucleonic sector

$$g_{iN}(\rho_B) = g_{iN}(\rho_0)h_i(x), \qquad h_i(x) = a_i\frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2} \quad i=\sigma,\omega,$$
$$g_{\rho N}(\rho_B) = g_{\rho N}(\rho_0)\exp[-a_\rho(x-1)], \quad i=\rho,(\pi-HF)$$

– DD-ME2 parametrization, Phys. Rev. C 71, 024312 (2005), Lalazissis, Vretrenar, Ring
– Similar to DD2 parametrizations (S. Typel)

	σ	ω	ρ
m_i [MeV]	550.1238	783.0000	763.0000
$g_{Ni}(\rho_0)$	10.5396	13.0189	3.6836
a_i	1.3881	1.3892	0.5647
b_i	1.0943	0.9240	—
c_i	1.7057	1.4620	—
d_i	0.4421	0.4775	—

$h_i(1)=1$, $h_i''(0)=0$ and $h_\sigma''(1)=h_\omega''(1)$, which reduce the number of free parameters to three in this sector.

Thermodynamics

Energy stress tensor for any generic field ϕ_i and its relevant components

$$T^{\mu\nu} = \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi_i)}\partial^\nu \phi_i - g^{\mu\nu}\mathscr{L}, \quad \to \quad \mathscr{E} \quad = \quad \langle T^{00}\rangle, \quad \mathscr{P} = \frac{1}{3}\sum_i \langle T^{ii}\rangle.$$

For the baryonic component

$$\mathscr{E}_B \quad = \quad \frac{\gamma_B}{2\pi^2}\int_0^{k_{F,B}} k^2 dk \left[T_B(k) + \frac{1}{2}V_B(k)\right],$$

$$T_B(k) \quad = \quad \hat{P}_B k_B + \hat{M}_B M_B, \quad V_B(k) = \hat{M}_B \Sigma_{S,B}(k) + \hat{P}_B \Sigma_{V,B}(k) - \Sigma_{0,B}(k),$$

with

$$\hat{P} = \vec{k}^*/E^* \quad \hat{M} = M^*/E^* \quad \vec{k}^* = \vec{k} + \hat{k}\Sigma_V \quad M^* = M + \Sigma_S.$$

Thermodynamic consistency requires:

$$\mathscr{P}_B = \rho_B^2 \frac{\partial \mathscr{E}_B}{\partial \rho_B}.$$

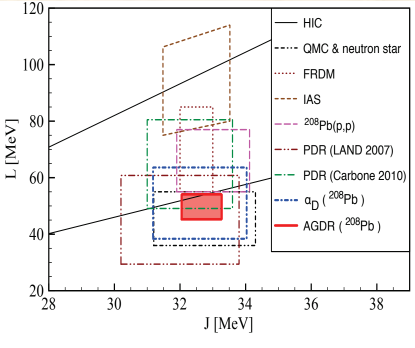
Taylor expansion of nuclear energy

$$\frac{E}{A}(\rho,\beta) = E_0 + E_{\rm sym}\beta^2 + L\beta^2\delta + \frac{1}{2!}(K_0 + K_{\rm sym}\beta^2)\delta^2 + \frac{1}{3!}Q_{\rm sat}\delta^3 + \mathscr{O}(\dots)$$

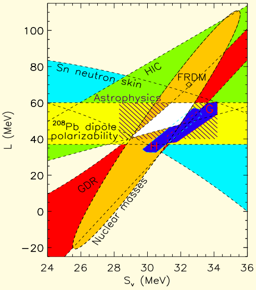
where $\beta = (n_n - n_p)/(n_n + n_p)$ and $\delta = (\rho - \rho_0)/3\rho_0$.

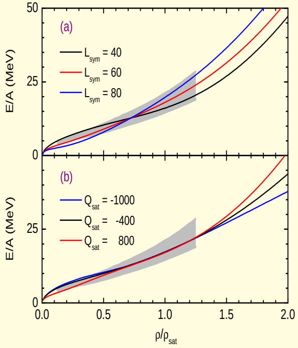
Consistency between the density functional and experiment

- saturation density $\rho_0 = 0.152\ \mathrm{fm}^{-3}$
- binding energy per nucleon $E/A = -16.14$ MeV,
- incompressibility $K_0 = 250.90$ MeV,
- symmetry energy $E_{\rm sym} = 32.30$ MeV,
- symmetry energy slope $L = 51.24$ MeV,
- symmetry incompressibility $K_{\rm sym} = -87.19$ MeV
- higher order $Q_{\rm sat} = 479$ $Q_{\rm sym} = 777$ MeV



Consistency between the density functional with experiment and ab initio theory



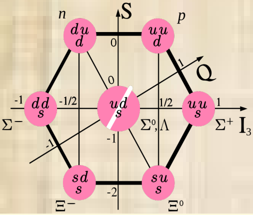


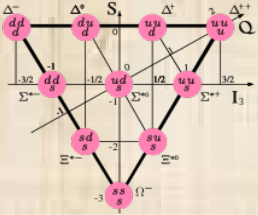
Lagrangian parameters can be replaced by *physical parameters (characteristics)* at saturation (Margueron, et al. 2018):

$\rho_0 = 0.152\ \mathrm{fm}^{-3}$, $E/A = -16.14$ MeV, $K_0 = 250.90$ MeV, (leading order)
$E_{\mathrm{sym}} = 32.30$ MeV, $L = 51.24$ MeV, (next-to-leading order)
$K_{\mathrm{sym}} = -87.19$ MeV (3rd order)

Beyond nucleons: Baryon octet $J^p = 1/2^+$ and baryon decuplet $J^p = 3/2^+$

Strangeness carrying baryons + resonances (nucleon excitations)





$R_{\alpha Y} = g_{\alpha Y}/g_{\alpha N}$ and $\kappa_{\alpha Y} = f_{\alpha Y}/g_{\alpha Y}$ for hyperons in SU(6) spin-flavor model

$R\backslash Y$	Λ	Σ	Ξ
$R_{\sigma Y}$	2/3	2/3	1/3
$R_{\sigma^* Y}$	$-\sqrt{2}/3$	$-\sqrt{2}/3$	$-2\sqrt{2}/3$
$R_{\omega Y}$	2/3	2/3	1/3
$\kappa_{\omega Y}$	-1	$1 + 2\kappa_{\omega N}$	$-2 - \kappa_{\omega N}$
$R_{\phi Y}$	$-\sqrt{2}/3$	$-\sqrt{2}/3$	$-2\sqrt{2}/3$
$\kappa_{\phi Y}$	$2 + 3\kappa_{\omega N}$	$-2 - \kappa_{\omega N}$	$1 + 2\kappa_{\omega N}$
$R_{\rho Y}$	0	2	1
$\kappa_{\rho Y}$	0	$-3/5 + (2/5)\kappa_{\rho N}$	$-6/5 - (1/5)\kappa_{\rho N}$
$f_{\pi Y}$	0	$2\alpha_{ps}$	$-(1/2)\alpha_{ps}$

$\alpha_{ps} = 0.40$. κ is the ratio of the tensor to vector couplings of the vector mesons.

The depth of hyperonic potentials in symmetric nuclear matter are used as a guide the range of hyperonic couplings:

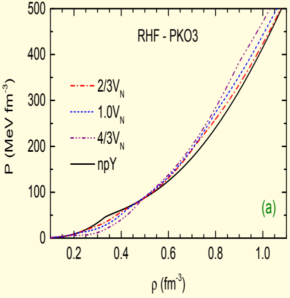
- Λ particle: $V_{\Lambda}^{(N)}(\rho_0) \simeq -30$ MeV
- Ξ particle: $V_{\Xi}^{(N)}(\rho_0) \simeq -14$ MeV
- Σ particle: $V_{\Xi}^{(N)}(\rho_0) \simeq +30$ MeV

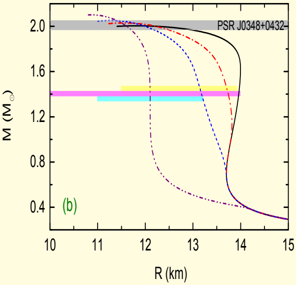
These ranges capture the most interesting regions of the parameter space of masses and radii.

The depth of Δ-potentials in symmetric nuclear matter are used as a guide the range the couplings:

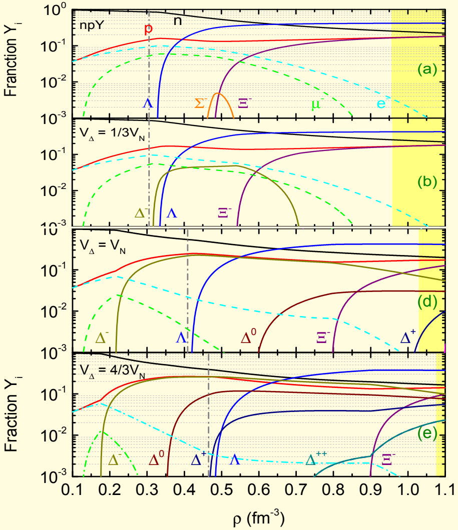
- Electron and pion scattering: $-30 \text{ MeV} + V_{\Delta}^{(N)}(\rho_0) \leq V_{\Delta}(\rho_0) \leq V_N(\rho_0)$
- Use instead $R_{m\Delta} = g_{m\Delta}/g_{mN}$ for which the typical range used is

$$R_{\rho\Delta} = 1, \quad 0.8 \leq R_{\omega\Delta} \leq 1.6, \quad R_{\sigma\Delta} = R_{\omega\Delta} \pm 0.2.$$

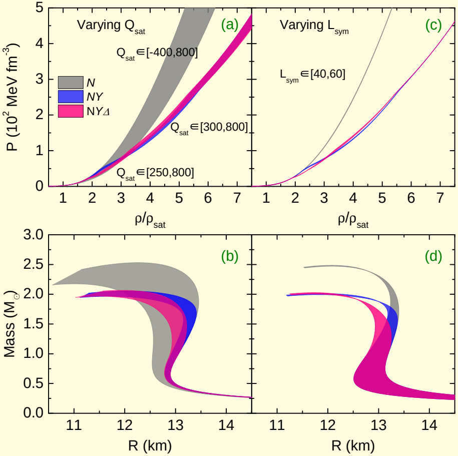




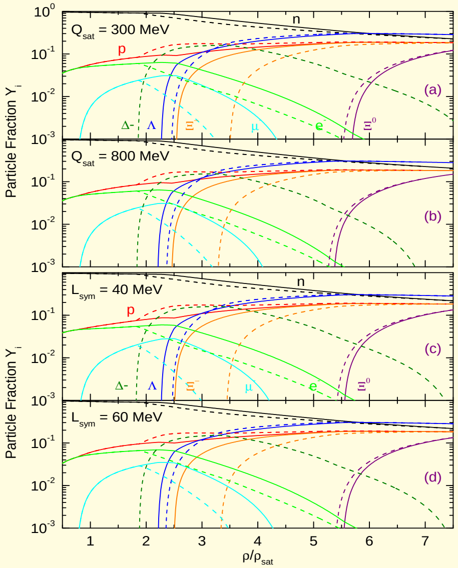
EoS and MR-relations for hyperonic and Δ-admixed models. Inclusion of Δ-resonances with different potential depths: strong reduction of the radius.



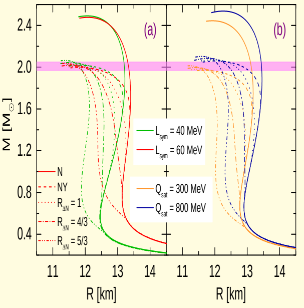
Large population ($\sim 20\%$) of heavy baryons. Λ population dominates at asymptotically large densities.

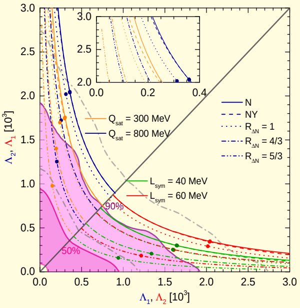


EoS models and MR relations for N, NY, and $NY\Delta$ compositions of stellar matter with the DDME2 parametrization. The bands are generated by varying the parameters Q_{sat} [MeV] (a, b) and L_{sym} [MeV] (c, d). The ranges of Q_{sat} and L_{sym} allowed by χEFT and maximum mass constraints are indicated in the figures.



Dependence of particle abundances on characteristics $L_{\rm sym}$ and $Q_{\rm sat}$.



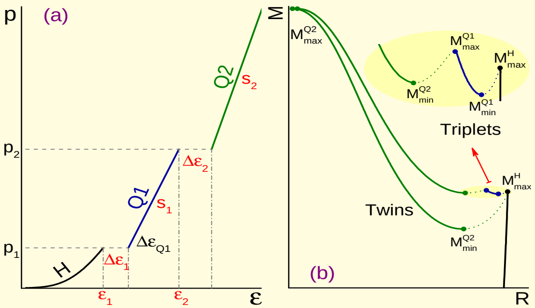


Tidal deformabilities

$$\Lambda = \frac{\lambda}{M^5} = \frac{2}{3}\frac{k_2}{C^5}, \qquad \lambda = \frac{2}{3}k_2R^5,$$

k_2 is tidal Love number, R star's radius, $C = M/R$ compactness for $NY\Delta$ matter and GW170817 constraints.

Consistency is achieved for low $L_{\rm sym}$ and $Q_{\rm sat}$ values and for heavy baryon compositions.



Left: EoS with two sequential phase transitions. *Right:* Mass-radius relationships, emergences of minima in the function $M(R)$.

Case when $NY\Delta$-matter makes a first order phase *sequential* transitions to various *generic new phases* (we had in mind phases of color superconducting phases).

$$p(\varepsilon) = \begin{cases} p_1, & \varepsilon_1 < \varepsilon < \varepsilon_1 + \Delta\varepsilon_1 \\ p_1 + s_1\left[\varepsilon - (\varepsilon_1 + \Delta\varepsilon_1)\right], & \varepsilon_1 + \Delta\varepsilon_1 < \varepsilon < \varepsilon_2 \\ p_2, & \varepsilon_2 < \varepsilon < \varepsilon_2 + \Delta\varepsilon_2 \\ p_2 + s_2\left[\varepsilon - (\varepsilon_2 + \Delta\varepsilon_2)\right], & \varepsilon > \varepsilon_2 + \Delta\varepsilon_2. \end{cases}$$

Density Functional Theory for Compact Stars

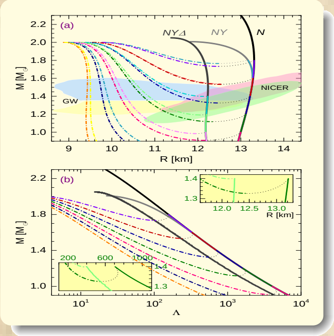
A Sedrakian

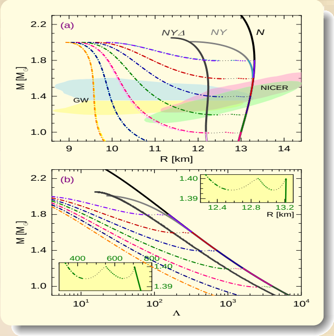
Equation of state of dense matter

Integral parameters of compact stars

Phase transition to quark matter

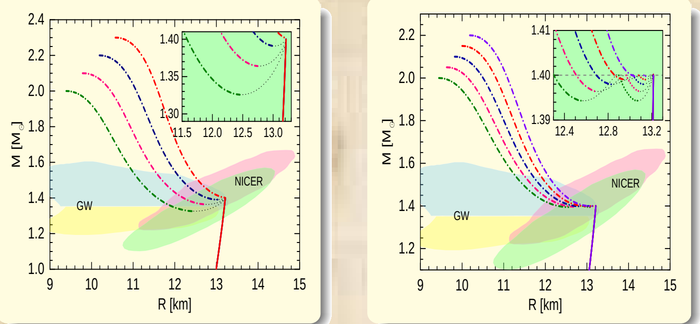
Rapidly rotating hybrid stars and GW190814



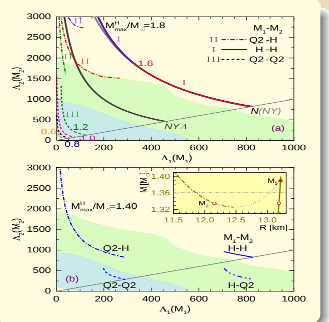


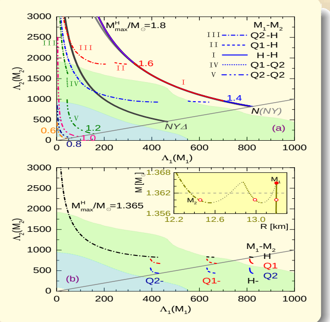
(a) Mass-radius relation for hybrid stars with a single (left) and double (right) phase transition(s), with three different hadronic envelopes: nucleonic (N), hyperonic (NY) and Δ-resonance–hyperon admixed ($NY\Delta$).
Each hybrid star branch bifurcates from a hadronic sequence at $M^{\rm H}_{\rm max}/M_\odot = 0.60$-$1.80$. (hyperons appear in the two cases: $M^{\rm H}_{\rm max}/M_\odot = 1.60$ and 1.80). In each case the maximum mass of the hybrid branch is fixed at $M^{\rm Q2}_{\rm max}/M_\odot = 2.00$.

(b) Mass-deformability relation for the configurations shown in (a) (only We NY or $NY\Delta$ envelopes). The inset has $M^{\rm H}_{\rm max}/M_\odot = 1.40$. The smaller radius (deformability) curve corresponds to $NY\Delta$ envelope stars, whereas the larger ones - to N-NY envelope stars.

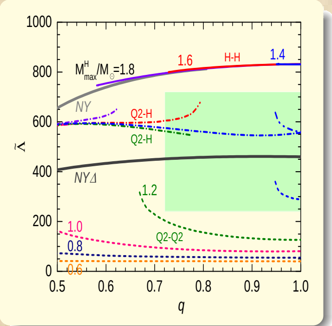


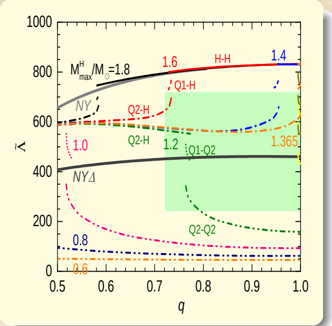
Mass-radius relation for hybrid EoS with single (left) and double (right) phase transitions and nucleonic envelope. The EoS are identified by the maximum mass $M^{\rm Q2}_{\rm max}/M_{\odot} = 2.00\text{-}2.20$, and the maximum mass of the hadronic star which is fixed at $M^{\rm H}_{\rm max}/M_{\odot} = 1.40$. The emergence of twin configurations is shown in the inset.





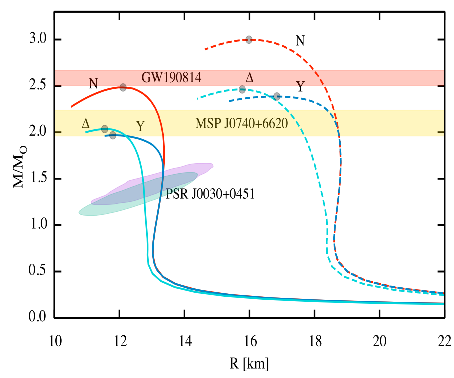
(a) Tidal deformabilities of compact objects with a single (left) and double (right) phase transition(s) for a fixed value of binary chirp mass $\mathcal{M} = 1.186M_\odot$. The are three types of pairs on the left and six pairs on the right (labelled by numerals I-VI). (b) The same as in (a) but for fixed$M^{\rm H}_{\rm max} = 1.40M_\odot$. The shaded regions correspond to the 50% and 90% credibility regions taken from the analysis of GW170817 within PhenomPNRT model. The inset shows the mass-radius relation around the phase transition region. The open circles (labeled M_2) are the masses of two possible companions for the star of mass M_1 (full circle) for a fixed value of binary chirp mass $\mathcal{M} = 1.186M_\odot$.



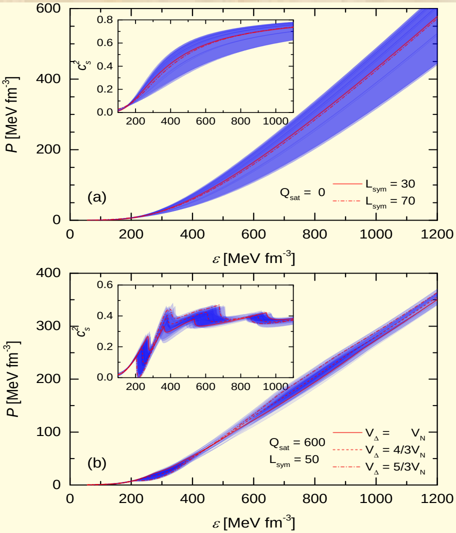


Mass weighted deformability vs. mass asymmetry for a binary system with fixed chirp mass $\mathcal{M} = 1.186 M_\odot$ predicted by a range of hybrid EoS with single phase transition and various values of $M^{\rm H}_{\rm max}$. The error shading indicates the constraints estimated from the GW170817 event and the electromagnetic transient AT2017gfo.

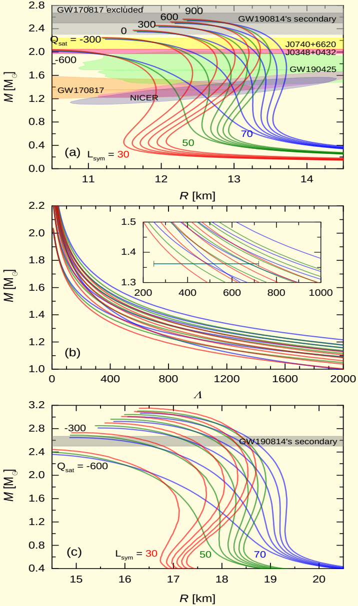
– GW190814 event: extreme mass asymmetric ratio created by a $22.2 - 24.3\,M_\odot$ black hole and a $2.50 - 2.67\,M_\odot$ compact object (no em counterpart).
– Light object's nature is enigmatic as it is in the mass gap $2.5\,M_\odot \lesssim M \lesssim 5\,M_\odot$ where no compact object had ever been observed before.



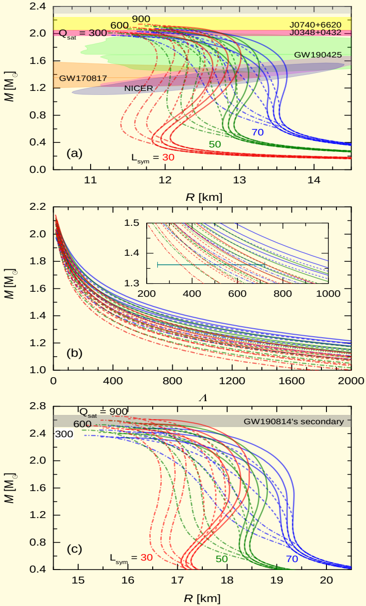
Solid curves – static solutions; dashed curves - maximally rotating (Keplerian) solutions.



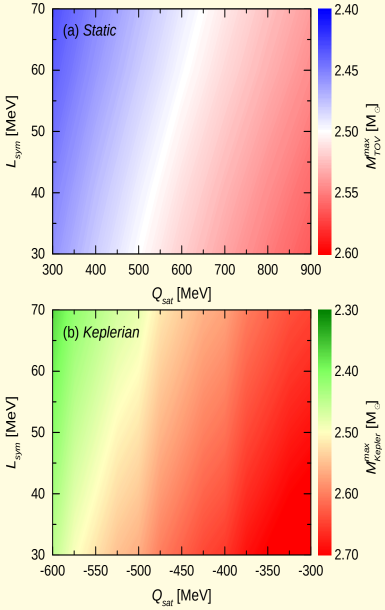
EoS and the corresponding speed-of-sound squared for (a) N and (b) $NY\Delta$ matter. In (a) $Q \in [-600, 900]$ and $L_{sym} \in [30, 70]$. EoS with $Q = 0, L_{sym} = 30$ and 70 are shown by solid and dash-dotted lines for illustration. In (b) $Q \in [300, 900]$, $_{sym} \in [30, 70]$ $V_D/V_N = 1, 4/3$ and $5/3$ EoSs with $Q = 600$, $L_{sym} = 50$ and three indicated values of V_D are shown for illustration.



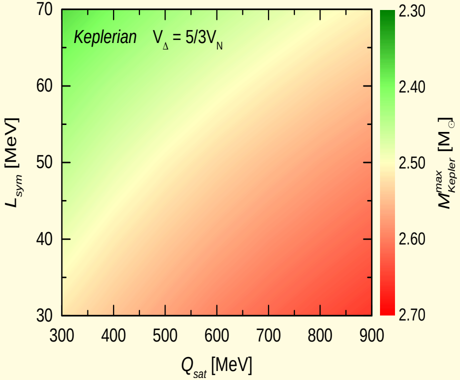
Mass-radius (a) mass-tidal deformability (b) for static N-stars. (c) Mass-radius for maximally rotating (Keplerian) sequences.



Mass-radius (a) mass-tidal deformability (b) for static $NY\Delta$-stars. (c) Mass-radius for maximally rotating (Keplerian) sequences.



Maximum masses of (a) static and (b) Keplerian N-stars.



Maximum masses of (a) static and (b) Keplerian $NY\Delta$-stars. The Δ potential $V_\Delta = 5/3 V_N$, the maximal value studied.

Further DFT developments (not covered in this talk)

- Hot compact stars, universal relations, limits on the maximum mass, S. Khadkikar, A. R. Raduta, M. Oertel, A. Sedrakian, arXiv:2102.00988, arXiv:2008.00213
- Hypernuclear Δ-admixed stars with anti-kaon condensation: V. B. Thapa, M. Sinha, J. J. Li, A. Sedrakian, arXiv:2102.08787
- Hypernuclear Δ-admixed stars in strong magnetic fields: V. B. Thapa, M. Sinha, J. J. Li, A.n Sedrakian, arXiv:2010.00981